A TEXTBOOK OF LIGHT

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A TEXTBOOK OF LIGHT

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PREFACE

This book is intended for the use of students who are already acquainted with the more elementary parts of the subject. To connect up with elementary light, geometrical optics is dealt with first, and physical optics follows. There is, of course, little reason why this order should not be reversed for advanced students, although experience shows it is easier, at the outset, to consider rays of light rather than waves. As is pointed out in Chapter XIII, rays of light are only considered for convenience, it being impossible to obtain a ray of light. The later chapters therefore deal with light travelling in the form of waves. Yet certain difficulties arise to-day in the use of the wave theory, and mention is made of these to show that this theory cannot be accepted without reservations. These difficulties are overcome by use of the quantum theory, which is, for this reason, briefly explained.

The usual sign convention has been adopted, and in the case of mirrors and lenses the incident light has been taken as proceeding from right to left, so that distances to the right of the mirror or lens are taken as positive, as in co-ordinate geometry. To aid the student in his own experiments brief notes are given on the simple methods of determining refractive indices and the constants of mirrors and lenses. The practical details of more advanced experiments of importance (*e.g.* Newton's rings) are given in full. Wherever possible brief references are made to applications of light in commerce. For further details the student should refer to more technical books.

Throughout the book the treatment is systematic, and in all cases the full mathematical theory has been given together with the experimental details.

Although designed for more general use, this book will

be found suitable for candidates preparing for the Higher School Certificate and the Intermediate Science examinations. The syllabuses for these are adequately covered, and the book further proceeds as far as University Scholarship standard. A large number of examples has been included, the majority having been set in recent public examinations. For permission to include these questions I am indebted to the Cambridge University Press, the London University Examination Board, the Northern Universities Joint Matriculation Board, the Oxford and Cambridge Schools Examination Board, and the Oxford University Press.

My thanks are due to Mr H. Cottam, B.Sc., for advice on certain points, and to Mr A. E. M. Bayliss, M.A., for assistance while reading the proofs. I am very grateful to Mr E. Evans, B.Sc., for the considerable help he has given, both in the preparation of the text and the examination of the proofs. Finally, I must acknowledge the assistance of the Electric Lamp Manufacturers Association on the subject of illumination.

L. R. M.

PREFACE TO THE SECOND EDITION

In this edition the whole of the sections relating to spherical mirrors and lenses has been rewritten so that a complete study of the subject may be made on any one of the three best-known sign conventions. It is only necessary for a student to use one convention, and the text is arranged in such a manner that it is quite easy to follow out the equations deduced for the convention chosen. Although the matter in connection with the other conventions should be disregarded at first, yet advanced students may profitably compare the merits of the three conventions.

Certain minor additions have also been made to the text, notably a reference to the new polarising material named Polaroid.

L. R. M.

July 1937.

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KEY TO ABBREVIATIONS USED AT THE END OF THE EXAMPLES

Lond. H.S.C. .	London University Higher School Certificate.
Lond. Inter.. .	London University Intermediate B.Sc.
N. . . .	Northern Universities Higher School Certificate.
O. & C. .	Oxford and Cambridge Joint Board Higher School Certificate.
Camb. Schol.	Open Scholarship, Cambridge University.
Ox. Schol. .	Open Scholarship, Oxford University.

CHAPTER I

RECTILINEAR PROPAGATION: PHOTOMETRY

THE subject of light may be conveniently divided into two parts—geometrical optics and physical optics. The former deals with certain of the simpler effects of light and with results obtained geometrically from the experimental fact that light travels in straight lines. Physical optics is concerned with the actual nature of light, and accounts for the departure from rectilinear propagation.

The fact that light travels in straight lines is a matter of everyday experience. It is impossible, in general, for light to travel round corners. The beam of light cast by the headlights of a car is bounded by straight lines, while any point on the edge of the shadow of an obstacle thrown by a source of light is always in a straight line with the obstacle and the source. It may thus be assumed as a fundamental fact that the propagation of light is rectilinear.

Assuming this, we may define a *ray* of light as any straight line in the direction in which the light travels, a *beam* of light as a bundle of rays, and a *pencil* of light

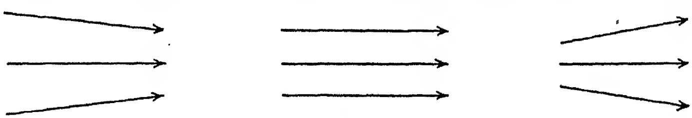


FIG. 1.

as an extremely narrow beam. A beam of light may be convergent, parallel or divergent. In a convergent beam the rays are approaching each other and moving towards a point; in a parallel beam the rays are all parallel; while in a divergent beam the rays become more separated as the beam travels along.

The Pin-hole Camera.—This instrument depends on the rectilinear propagation of light. If a luminous object is

placed on one side of an opaque screen containing a small pin-hole, and a white screen is on the other side, then an inverted image of the object is formed on the latter. The image is always sharp and clear, irrespective of the distances between object and pin-hole, or pin-hole and screen. The

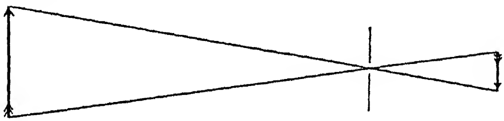


Fig. 2.

reason for this is that every point on the object sends out rays of light, of which only a few pass through the pin-hole and reach the screen. Thus an image of every point of the object is formed on the screen in the manner indicated. If the hole is enlarged the effect is the same as having a number of pin-holes close together. An image is formed by rays passing through each pin-hole, and the images overlapping cause the resultant image to be blurred. If a photographic plate is substituted for the screen a photograph of the object can be obtained. It will be noticed, however, that a small pin-hole is necessary in order for a sharp image to be formed, and with a small pin-hole the image is faint, necessitating a very long exposure. The pin-hole camera thus has the advantage that no focussing is required, but the serious disadvantage that a long exposure is necessary.

Shadows.—The shadow of an object thrown by a point source of light has always a sharp boundary. This is an experimental fact which leads to the conclusion that light

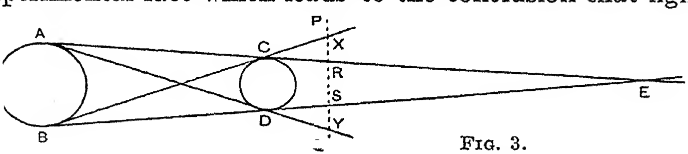


Fig. 3.

travels in straight lines. If the source of light is an extended one, a shadow is thrown by every point of the

source and the shadows partly overlap. Let AB and CD represent the source of light and the obstacle respectively, both being spherical. If we draw lines AC and BD from the extremities of AB and just touching CD, these lines meet at E, so that within the cone CDE there is complete darkness. This cone is known as the shadow cone of CD thrown by AB, and if a screen PQ is placed between CD and E this shadow will cause a central dark spot, RS. Surrounding this will be a ring of shadow XY, which will get lighter the further from RS. This ring is partly illuminated by AB, the illumination depending on the distance from RS. At X and Y the whole of AB is visible, so that for parts of the screen surrounding XY there is no shadow. The whole shadow is thus made up of a black spot RS, which is called the *umbra*, surrounded by the ring of increasing brightness XY, which is called the *penumbra*. It must be noticed that there is no sharp boundary between the umbra and penumbra, since the darkness merely decreases in intensity as we pass from RS outwards.

The application to eclipses may easily be seen. If the sun is represented by AB and the moon by CD, a shadow cone is cast as in the diagram. If the earth passes through this cone CDE there is a total eclipse of the sun for all parts within the cone, while for all parts within the penumbra there is a partial eclipse.

Photometry

Photometers are instruments for measuring the relative illuminating powers of sources of light. Photometry is thus the science of comparing and measuring the powers of illuminants, and of finding the intensities of illumination of surfaces. Before considering the different types of photometers it is necessary for us to know the exact meaning of the term illuminating power and also to examine the standards of illumination.

Consider a sphere of unit radius described round a point source as centre. Each unit area of the surface of the

sphere will receive the same quantity of light in the same time. The quantity falling on unit area in unit time is termed the illuminating power of the source. The definition of the term *illuminating power* is thus the quantity (or energy) of light which in unit time falls normally on unit area at unit distance.

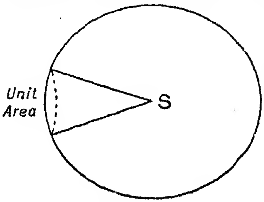


Fig. 4.

It is obvious that the quantity of light falling on unit area depends on the distance from the source. This leads to the term *intensity of illumination* of a surface, this being defined as the quantity or energy of light which falls on unit area in unit time.

The Inverse Square Law.—Consider a point source S of illuminating power I at the centre of two spheres which have radii r_1 and r_2. By the definition of illuminating power the total quantity of light emitted must be 4πI since the area of surface of a unit sphere is 4π. The intensity of illumination of the surface of the inner sphere is therefore

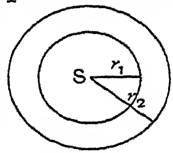


Fig. 5.

$$\frac{4\pi I}{4\pi r_1^2} (= F_1).$$

If the inner sphere is removed all the light falls on the other sphere, the intensity of illumination being $\frac{4\pi I}{4\pi r_2^2} (= F_2)$.

Hence
$$\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2},$$

or the intensity of illumination is inversely proportional to the square of the distance from the source.

Also we have

$$\text{Intensity of illumination} = \frac{\text{Illuminating power}}{(\text{Distance})^2} \text{ since } F_1 = \frac{I}{r_1^2}.$$

If the light does not fall normally on the surface the intensity of illumination is reduced. Suppose the surface XY

has an area A, and suppose the normal to the surface makes an angle θ with the direction of the incident light. All the light falling on the surface passes through an area XZ which is normal to the central ray of light. Now the area of XZ is A $\cos\theta$. The quantity of light falling on this area is thus $\dfrac{\mathrm{IA}\cos\theta}{d^2}$

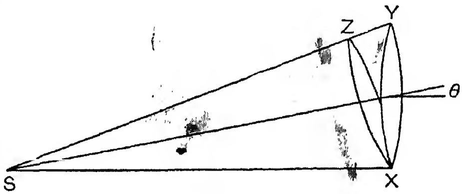


FIG. 6.

Since the whole of this light illuminates the surface XY which has area A, the intensity of illumination of XY is $\dfrac{\mathrm{I}\cos\theta}{d^2}$. Thus, as the obliquity increases, $\cos\theta$ decreases, and the intensity of illumination gets smaller and smaller in proportion. Hence, in general, $\mathrm{F}=\dfrac{\mathrm{I}\cos\theta}{d^2}$.

In comparing illuminating powers the intensities of illumination produced by the sources are made equal, the obliquity of the screen or area being the same to each source. Then

$$\frac{\mathrm{I}_1\cos\theta}{d_1^2}=\frac{\mathrm{I}_2\cos\theta}{d_2^2}$$

or

$$\frac{\mathrm{I}_1}{\mathrm{I}_2}=\frac{d_1^2}{d_2^2}.$$

Standards of Illumination.—The unit of illuminating power is the illuminating power of a standard candle, which is a sperm candle burning 120 grains of wax per hour. Illuminating power is thus expressed in candle-power. This standard is a most unsatisfactory one, and several other standards have been suggested, as follows:—

(1) The pentane lamp, 10 candle-power, designed by Vernon Harcourt, and used chiefly in this country.

(2) The amyl acetate lamp, 0·9 candle-power, designed

by Hefner, and used as the unit in Germany. This is a very simple standard, but has disadvantages in that the flame has a reddish tint and that the illuminating power is somewhat low.

(3) The platinum standard, suggested by Violle, this being the light emitted normally by one square centimetre of platinum at its melting point. The manipulation required in using this standard renders it almost useless for most practical purposes.

(4) A special form of incandescent electric lamp suggested by Fleming as a secondary standard. The voltage must be kept constant and, subject to this condition, the light emitted is constant. After some use, however, the filament changes slightly and the glass blackens, altering the illuminating power of the lamp.

✓*Photometers.*—It must be remembered that the inverse square law holds only for point sources, so that the distance between the source and the photometer must be large compared with the length or width of the source. As a general rule photometers depend on the adjustment of two similar surfaces to equal brightness, the surfaces being seen by the eye simultaneously or successively. One of the difficulties in photometry is that the apparent brightness of the source depends on its colour, thus making it difficult to compare lights of different tint.

(1) *The Rumford Photometer.*—Shadows of a rod R, mounted vertically, are thrown on a screen by the two sources of light, S_1, S_2. The distances of the sources from

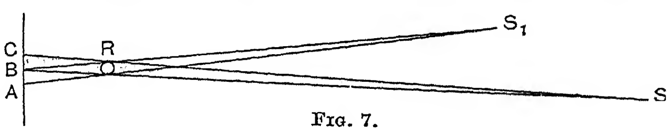


FIG. 7.

the screen are adjusted until the shadows appear equally dark. The rod is also moved so that the shadows are just made to touch, comparison of the darkness of the shadows then being much easier. The distance of R from the

screen should not affect the accuracy of the experiment, but if the lights are not point sources the edges of the shadows will not be sharp unless the rod is near to the screen and S_1, S_2 some distance away.

Now the part AB of the screen is illuminated only by S_2, while BC is illuminated only by S_1. Since the shadows are equally dark the intensities of illumination at B due to each source are equal, so that if I_1 and I_2 denote the illuminating powers of S_1 and S_2,

$$\frac{I_1}{S_1B^2} = \frac{I_2}{S_2B^2}.$$

(2) *The Bunsen Photometer.*—In its simplest form this photometer consists of a piece of white paper with a grease spot near its centre. The paper scatters most of the light falling on it, but grease renders paper translucent so that some of the light falling on the grease spot is transmitted. If the intensities of illumination on the two sides of the paper are equal then equal quantities of light are transmitted by the grease spot in each direction, and consequently the spot is indistinguishable from the surrounding paper. Thus in comparing two light sources the paper containing the grease spot is mounted between the lights and moved until the spot seems to disappear. In practice it is found that the grease spot does not disappear at the same position when viewed from one side of the paper as from the other. This is due to the loss of light by scattering at the surface of the paper surrounding the grease spot. The effect is still more noticeable if the sources of light differ in colour. It is thus advisable to view the two sides of the paper at

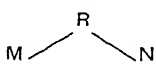


FIG. 8.

the same time and to adjust the position so that the appearance of each side is similar. In order to do this two plane mirrors, MR, NR, are placed at equal angles to the

plane of the paper, and both images are examined together. After taking a reading the positions of the lights should be interchanged and a further reading taken. This eliminates any error which may be caused by stray light falling on one side of the paper.

(3) *The Wedge Photometer.*—A wedge, the sides of which are covered with diffusing paper, is placed between the two sources of light S_1, S_2 so that the faces make equal angles with the line S_1S_2. The light diffused from each

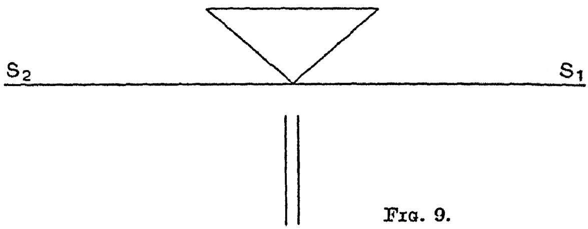


FIG. 9.

face is observed in a direction perpendicular to S_1S_2, and the position of the wedge is such that each face appears equally bright. The illuminating powers of the sources are thus proportional to the squares of the respective distances from the edge of the wedge. It is advisable to view the faces of the wedge through a cardboard tube blackened on the inside, and to adjust the position of one of the sources until it is impossible to distinguish the edge between the faces of the wedge.

✓ (4) *The Joly Photometer.*—A sheet of tinfoil, T, is placed between two equal blocks of clear paraffin wax, A and B. The lights to be compared are placed on opposite sides of the photometer, the position of which is adjusted until each block appears equally bright when observed in a direction at right angles to S_1S_2. The light from each source is

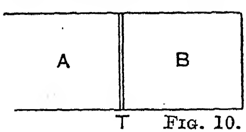


FIG. 10.

scattered inside the respective blocks, and when the intensities are equal the illuminating powers of the sources

are proportional to the squares of their distances from the respective faces of the photometer. The paraffin wax blocks may be replaced by ground glass slabs, the effect being exactly the same.

(5) *The Lummer-Brodhun Photometer.*—This photometer consists of a diffusing screen, D, usually a slab of magnesium carbonate, placed between the sources and at right angles

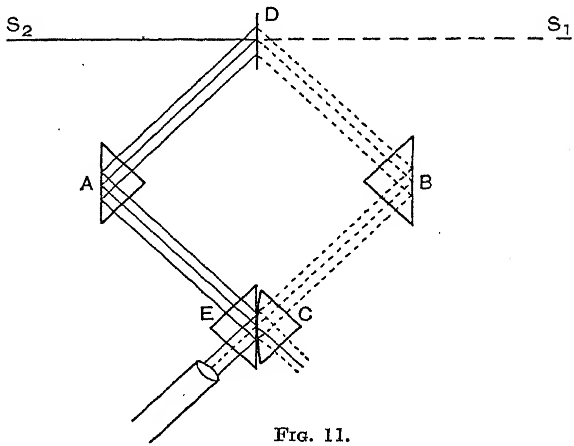


FIG. 11.

to the line joining them. Some of the diffused light falls on two prisms, A and B, which are arranged to reflect the light into a double prism as shown. This consists of two right-angled prisms, E and C, the hypotenuse of E being plane, while that of C is curved. The two prisms are placed together, and are in contact over a small central area only. Thus, of the light reaching C from the source S_1, the central part is transmitted into E while the outer parts are reflected at the hypotenuse. In a similar manner light falling on the outer parts of the hypotenuse of E is reflected, while that falling on the central part is transmitted through prism C.

Thus the field of view of the eyepiece consists of a central

part illuminated by light from S_1 surrounded by portions which are illuminated by light from S_2. One of the sources is moved towards or away from D until the whole field of view appears equally bright. Then $\frac{I_1}{S_1D^2}=\frac{I_2}{S_2D^2}$, where I_1, I_2 represent the illuminating powers of S_1 and S_2 respectively. In order to eliminate any effects due to differences in the surfaces of the screen the sources are interchanged and a further reading taken. It will be noticed that there is considerable resemblance between this photometer and Bunsen's grease spot photometer, although the latter is not nearly so accurate. The Lummer-Brodhun is, in fact, probably the most accurate of all photometers, provided the lights are of the same colour.

(6) *The Flicker Photometer.*—A grave disadvantage of all the photometers considered so far is the lack of accuracy which occurs when the sources are of different colour. In the flicker photometer this difficulty does not arise, since here two screens—illuminated by the two sources respectively—are viewed in rapid succession so that the colours merge. This photometer is very simple in construction, consisting of two screens, A and B, one of these being fixed, the other one, A, being in the form of a circle divided into eight equal sectors, of which alternate ones are removed. This screen is capable of rapid rotation, by means of an electric motor, about an axis through its

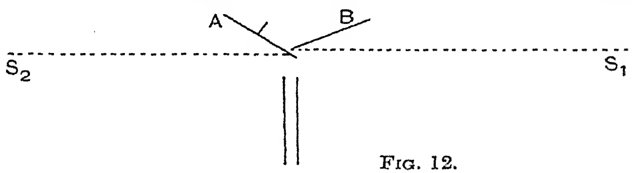


FIG. 12.

centre. A and B are arranged so that they make equal angles, usually 30°, with the line joining S_1 and S_2. They also overlap slightly, so that when viewed through a tube at right angles to S_1S_2 equal areas of each screen are seen

in turn and a flickering sensation is produced. When the intensities of illumination are equal the sensation of flickering disappears entirely. The illuminating powers are then proportional to the squares of the distances of the sources from the parts of the screens seen through the observation tube.

If the colours of the sources are different the colours merge, but the flickering still appears until the intensities are made equal.

Units of Illuminating Power and Intensity of Illumination. —The unit of illuminating power is the *lumen*, which is the light energy falling per second on unit area at unit distance from a standard candle.

The intensity of illumination at a distance of one metre from a standard candle is termed one metre-candle or one *lux*. Since intensity of illumination $=\dfrac{\text{illuminating power}}{(\text{distance})^2}$, 1 lux = 1 lumen per square metre.

At a distance of one centimetre from a standard candle the intensity of illumination is 10,000 lux, or 1 lumen per square centimetre, and is termed the *phot*.

On the F.P.S. scale the unit is the *foot-candle*. A standard candle produces an intensity of illumination of one foot-candle on a screen at a distance of one foot.

Other units used are the mean horizontal candle-power and the mean spherical candle-power. Sources of light do not give out light equally in all directions, so that if a photometric measurement of the candle-power of a lamp in a certain direction is made, this result may be quite different from the candle-power found in another direction. The average value for the candle-power found at points in a horizontal plane is therefore obtained, the axis of the lamp being vertical. The illuminating power of the lamp is then called the *mean horizontal candle-power* (m.h.c.p.). This unit is not always satisfactory, since in some cases —such as street lighting—the lamp is required to emit light in directions below the horizontal plane through the

lamp. A more general unit in which to express the illuminating power of a lamp without reference to any special direction is thus desirable. This unit is the *mean spherical candle-power* (m.s.c.p.), and is the measure of the total light output of a lamp. It may be measured by determining the candle-powers in a number of equally distributed directions from the source and taking the average value. If a point source of unit candle-power is at the centre of a sphere of unit radius, the surface area of the sphere is 4π, so that by the definition of the lumen the total illumination of the source can be expressed as 4π lumens. Consequently the total illumination of a lamp, measured in lumens, equals the mean spherical candle-power multiplied by 4π.

Experiments in Photometry.—An interesting experiment is to find how the efficiency of an electric lamp varies with the wattage applied to it. A "standard" electric lamp is set up at a fixed distance from the photometer, the lamp under test being placed on the opposite side. The current and voltage applied to the test lamp are read off and the distance from the photometer measured when the intensities at the photometer are equal. The illuminating power of the test lamp is then found in terms of that of the standard, so that if d and D are the respective distances

$$\frac{I}{I_0}=\frac{d^2}{D^2}$$

where I_0 refers to the standard.

The efficiency is expressed in terms of candle-power per watt, and so a measure of the efficiency E may be obtained by dividing I/I_0 by the wattage W. Further readings are taken for different values of the wattage. Now the relation between E and W is $E = kW^x$ approximately, where k and x are constants. This formula leads to

$$\log_{10} E = \log_{10} k + x \log_{10} W.$$

If a graph of $\log_{10} E$ against $\log_{10} W$ is drawn (fig. 13), the

slope gives the value of x. So it is possible to find on what power of the wattage the efficiency of a lamp depends.

Another useful experiment is to find the percentage of light transmitted by a glass plate. If the flicker photometer is used a comparison of the intensities of the light transmitted by plates of different colour may be made. This fraction of the light transmitted is sometimes called the transmission factor of the plate.

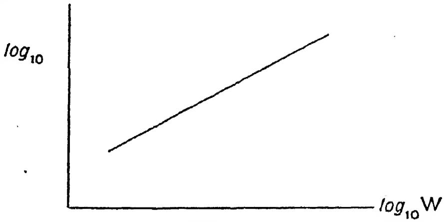


FIG. 13.

The photometer is set up as usual between any two sources, and is adjusted so that the intensities of illumination are equal. The plate is placed between one source and the photometer. This necessitates this source being moved nearer to the photometer to regain equal intensities. If the initial distance of the source from the photometer is x and the final distance is y, then the initial intensity of illumination is I/x^2 where I is the illuminating power, while the final intensity is pI/y^2. These are equal since the other source remains at a fixed distance from the photometer. Hence the percentage of light transmitted $=100p=100\frac{y^2}{x^2}$, where p is the fraction of the light transmitted by the glass plate.

Applications of Photometry.—The subject of photometry is rapidly becoming of greater importance since it is now realised that the efficiency of persons working in artificial light depends considerably on the intensity of illumination. For general purposes such as reading or writing an intensity of illumination of about 4 foot-candles is required, but for more delicate work it is necessary to have an intensity of about 12 foot-candles. Since the illumination of a small area is frequently required, portable photometers

have been constructed and these are so calibrated that the illumination can be read off directly in foot-candles. These instruments may either be of the grease-spot type in which the illuminations on each side of a translucent screen are made equal, or be of a type employing photo-electric cells. In the former an electric bulb is joined to a variable resistance, battery and voltmeter, and when using this instrument the resistance is adjusted so that a standard voltage is registered. The bulb illuminates a series of circular translucent screens which are in a line going away from the bulb. The illumination thus varies, being greatest on the screen nearest the bulb and gradually diminishing to the screen farthest away. When this photometer is placed on any plane where the illumination is required, the screens near the bulb generally appear bright, while those farther away appear dark. One of the screens between appears neither light nor dark since the illumination due to the bulb here is just balanced by the illumination at that place. At each of the screens the intensity of illumination due to the bulb is marked in foot-candles, and so the illumination at any place in a room may be read off.

In the other type use is made of the photo-electric effect. When a metal plate is illuminated it is found to emit electrons, and by measuring the electronic current it is possible to obtain a measure of the illumination. For most metals the photo-electric effect is produced by ultra-violet rays, but the alkali metals, which exhibit this effect to a much greater extent than other metals, respond to all visible wave-lengths, and so are suitable for use in photometry. This type of photometer thus consists of a searching unit containing a photo-electric cell, which is joined to a micro-ammeter. This micro-ammeter is calibrated in foot-candles so that as the searching unit is moved from place to place the illumination is read off directly.

A modification of this photo-electric photometer has been employed for measuring the optical densities of

photographs of spectra. A beam of light is split up so that part travels to a photo-electric cell through the photographic plate (the negative), and the other part travels to the cell *via* an optical wedge. By means of a shutter only one beam is allowed to fall on the photo-electric cell at a time. The thickness of the wedge in the path of the light is so adjusted that each beam has the same effect on the photo-electric cell, and there is no change in the current produced when the shutter is moved so that first the one and then the other beam falls on the cell. So the optical density of that portion of the photograph of the spectrum equals the optical density of the wedge at the point through which the light passes.

There are many other instances of the use of photo-electric cells in connection with photometry. Such cells are employed to work a relay and to switch on a lamp as soon as the general illumination falls below a certain value. The photo-electric cell is not itself exposed to the lamp, and as soon as the general illumination of the cell reaches the required standard the lamp is switched out by the relay. In talking films and also in certain gramophone records containing perforations, the variations in the light intensity falling on a photo-electric cell have been employed in order to reproduce sound.

The *Colorimeter* is another photometric instrument, and is employed for determining the amount of a given substance in a solution. A beam of light passes through a circular aperture and is then divided into two beams. The two liquids—the standard and the one under test—are contained in tubes, and one beam passes through the first liquid and the other through the second liquid. In travelling through these liquids absorption takes place, this absorption depending on the concentration of the liquid and on the length of path through the liquid. The beams are brought together by means of prisms, and are viewed through a microscope. The length of path through one of the liquids is then adjusted until the two beams are

equally bright and the image seen in the microscope is uniformly illuminated. The relationship between the concentration (c) and the length of path through the liquid (l) is

$$c \propto \frac{1}{l},$$

so that the concentration of the liquid under examination is easily obtained.

EXAMPLES ON CHAPTER I

1. Explain fully how you would use Rumford's shadow photometer for comparing the illuminating powers of two sources of light. Why is it that this experiment need not be performed in a dark room?

2. Define the terms illuminating power, intensity of illumination and lumen. Explain how you could verify the inverse square law by use of Bunsen's photometer.

3. A grease spot photometer is placed midway between two lamps of equal illuminating powers. When a sheet of glass is placed between the screen and one lamp, it is found that the other lamp must be moved away by 20 cm. in order to restore a balance. An exactly similar sheet of glass is now added to the first sheet. How much farther must the lamp be moved for a balance if the initial distance between the lamps is 120 cm.? Find also the fraction of the light transmitted by one of the sheets of glass.

4. Define foot-candle and explain what is meant by the candle-power of a source of light.

Two lamps are situated 50 cm. and 70 cm. respectively from a flicker photometer when the sensation of flicker disappears. A sheet of red glass is placed between the photometer and the second lamp which now has to be moved 40 cm. nearer for a balance to be obtained. Find the percentage of the light absorbed by the red glass.

5. The intensity of illumination of a screen, illuminated by a lamp 5 feet away, is 10 foot-candles. Calculate the intensities as the screen is rotated through (a) 30°, (b) 60°, and find in each case how much nearer the lamp should be brought in order to maintain a constant intensity of illumination.

6. An electric lamp of candle-power 40 is placed at the focus of a convex lens which has a focal length of 5 cm. What is the intensity of illumination of a screen on which the parallel beam of light falls?

7. A screen is illuminated by a lamp which is 60 cm. away. A sheet of glass is placed between the screen and the lamp which has to be moved 6 cm. nearer to maintain the same intensity of illumination as before. What fraction of the light is absorbed by the sheet of glass?

8. A point source of light is 50 cm. from a screen, and is arranged to be at the focus of a concave mirror which reflects light on to the screen. The focal length of the mirror is 6 cm. Compare the intensities of illumination of the screen when the mirror is present and when the mirror is removed.

9. Describe the apparatus with which you would compare the candle-powers of two sources of light. Explain how you would use it to test the statement that a given glass plate transmits only 40 per cent. of the incident light. (Lond. Inter.)

10. Describe the shadow and one other form of photometer, explaining how the observations are used in getting the final results.

A paper screen has a thin rod 20 cm. in front of it. Beyond the rod, 120 and 180 cm. respectively from the screen, are two sources of light, which produce on the screen shadows of the rod of equal intensities. Calculate the ratios of the widths of the shadows and also the candle-powers of the lights. (Lond. Inter.)

11. Describe a method of comparing the intensities of light from two sources, and explain how to find the variations of the intensity of an electric light due to changes in the current. (Lond. Inter.)

12. Describe a method of comparing the illuminating powers of two sources of light, and discuss its limitations.

It is found that the illumination produced by a source A on a photometer screen is balanced by a source B at a distance of 60 cm. from the screen. A piece of smoked glass is placed in front of A, and in order to produce a match B has to be moved 15 cm. farther away. Calculate the fraction of light transmitted by the glass.

(O. & C.)

13. Explain the meaning of the terms: mean spherical candle-power, lumen, foot-candle.

Describe a modern form of photometer, showing how it is an improvement on a simple Bunsen or Rumford photometer.

How would you determine the fraction of the incident light transmitted by a sheet of glass? (N.)

14. Describe how you would compare the candle-powers of two lamps, pointing out the precautions you would take to obtain as accurate a result as possible.

A point source of 500 C.P. is suspended 20 ft. above a horizontal road. What is the intensity of the illumination (*a*) at a point P on the road vertically below the lamp, (*b*) at points on the road 30 ft. from P? In what units are your results expressed? (N.)

15. Describe, with the necessary theory, an accurate method whereby the intensities of illumination of two sources of light may be compared. Suggest a form of apparatus suitable for comparing the reflecting powers of two plane mirrors. (O. & C.)

16. Give an account of the theory and mode of action of an accurate form of photometer.

How far is it possible to compare the intensities of two sources of light differing in colour? (O. & C.)

CHAPTER II

REFLECTION AND REFRACTION AT PLANE SURFACES

When light falls on any substance it may suffer

(1) absorption, (2) transmission, (3) reflection.

In many cases all these occur simultaneously to varying extents. A substance which transmits none of the light is termed opaque, while one which transmits the light may be either transparent or translucent. In the former case an object can be seen distinctly through the medium (*e.g.* clear glass), while in the latter only an indistinct image is observed (*e.g.* ground glass or fog).

Reflection.—Reflection at a surface may be either regular or irregular. All substances, except dead black ones, scatter the light which falls on them, and thus appear visible. Regular reflection occurs only at smooth or polished surfaces. The laws relating to regular reflection at plane surfaces will now be considered.

The ray of light falling on the polished surface is known as the incident ray, the point where the incident ray meets the mirror or reflecting surface as the point of incidence, and the ray leaving the mirror as the reflected ray. The perpendicular to the mirror at the point of incidence is called the normal, and the angles made by this line with the incident and reflected rays are the angles of incidence and reflection respectively. Then the two *laws of reflection* are:

(1) The angle of incidence equals the angle of reflection.

(2) The incident ray, the normal at the point of incidence and the reflected ray are all in the same plane.

They may be verified as follows:—

Place the mirror on edge on a sheet of paper fixed to a drawing board and draw a line MN on the paper along

the edge of the reflecting surface. Draw any other line AB making an angle ABM with the mirror. View the reflection of AB in the mirror and place a straight edge on the paper so that it appears in line with the image of AB. Remove the mirror, mark the position of the straight edge PB, and draw the normal BC at B. Then AB is the incident ray, BC is the normal and BP is the reflected ray.

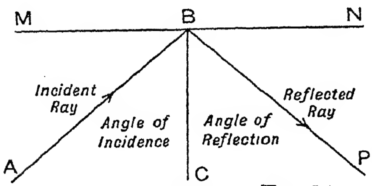


FIG. 14.

These are all in the plane of the paper, thus verifying the second law. Also the angles ABC and PBC will be found to be equal by measurement, and if the experiment is repeated for varying angles of incidence it is always found that these angles are equal to the corresponding angles of reflection. The experiment may be carried out more rapidly if an electric lamp is used and two narrow slits are placed in front of it. This produces a pencil of light, and the incident and reflected pencils can be seen easily.

Position of the Image formed by Reflection at a Plane Mirror.—Let MN be the mirror, P a point object in front

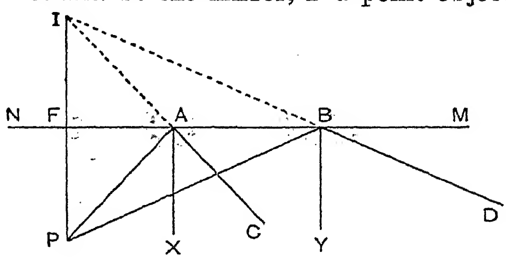


FIG. 15.

of it, and PA, PB be any two rays to the mirror. The corresponding reflected rays are AC and BD, and these appear to diverge from a point I behind the mirror. The point I is called the image of P. Join I to P, cutting

MN at F. Drawing normals AX and BY to the mirror,

we have $P\hat{A}X = C\hat{A}X$ and $P\hat{B}Y = D\hat{B}Y$,
whence $P\hat{A}F = C\hat{A}M$ and $P\hat{B}A = D\hat{B}M$.
But $C\hat{A}M = I\hat{A}F$ and $D\hat{B}M = I\hat{B}A$,
$\therefore$ $P\hat{A}F = I\hat{A}F$ and $P\hat{B}A = I\hat{B}A$.

Then the triangles PAB and IAB must be equal since $P\hat{B}A = I\hat{B}A$, AB is common, and $P\hat{A}B = I\hat{A}B$ (supplements of equal angles). Consequently $PA = AI$. Then in the triangles PAF and IAF we have $P\hat{A}F = I\hat{A}F$, AF is common, and $PA = AI$, so that these triangles are congruent. Hence $PF = FI$ and $P\hat{F}A = I\hat{F}A$, but since these angles are adjacent, each equals 1 right angle. Thus the line joining object to image is bisected at right angles by the mirror.

Real and Virtual Images.—In the case just considered the reflected rays appeared to come from a point I behind the mirror, so that P appeared to be at this point. Hence the image of P is situated at I. Images are divided into two classes—real and virtual. An image is real when the rays of light actually pass through the place where the image is, and virtual when the rays only appear to pass through that place. Thus a real image may be formed on a screen while a virtual one cannot. For plane mirrors the images are always behind the reflecting surfaces and are virtual.

Use of the Principle of Parallax to Determine the Position of Images.—The apparent motion of one object relative to another as the position of the eye is varied is termed parallax. Experience shows that the object more distant from the eye always appears to move in the same direction as the eye relative to the other object. If the two objects coïncide in position there is, of course, no apparent relative motion, and consequently there is absence of parallax.

A plane mirror is placed on edge, the reflecting surface being a straight line MN (fig. 16), and a pin P is placed in front of the mirror. A second pin Q is placed any distance behind the mirror, and the top of this pin, seen

over the mirror, and the image of P, seen by reflection, are observed. The eye is moved from side to side, and the position of Q changed until Q and the image appear in a straight line for all positions of the eye, *i.e.* there is no parallax between them. Then Q and the image of P must coincide in position, showing where the image is situated. Measurement shows that a line joining PQ is bisected at right angles by the line MN.

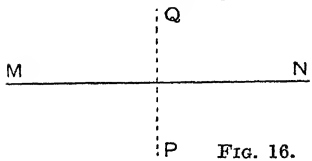


FIG. 16.

Image of an Elongated Object. Lateral Inversion.—If an extended object PQ is used instead of a point source, each point on the object forms an image in accordance with the above result. Thus R is the image of P, S the image of Q,

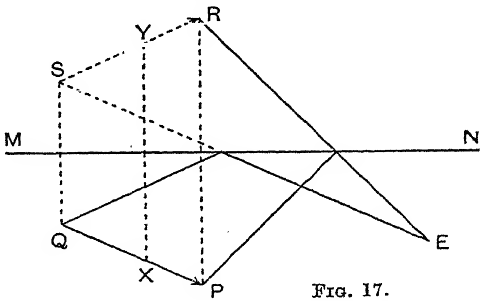


FIG. 17.

and Y of X, where PR, QS and XY are all bisected at right angles by the line MN. If the eye is situated at E it will be noticed that Q is on the right of the object, while S, the image of Q, appears on the left of the image RS. A similar case occurs if a piece of blotting paper, which has been used to blot some writing, is observed by reflection at a mirror. The writing can then be read directly. This reversal by reflection is known as lateral inversion.

Rotation of a Plane Mirror.—When a mirror is rotated a beam of light reflected by the mirror is rotated also. Let

PO denote a ray of light incident on a mirror MN, the reflected ray being OQ, and OC being the normal. If the

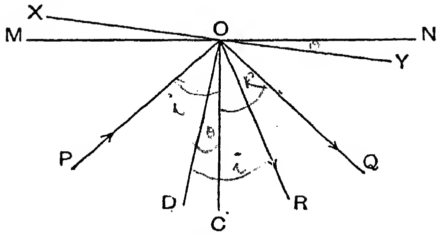


FIG. 18.

mirror is now rotated about O into the position XY, the reflected ray will travel along OR, and the normal will be OD.

Then $\quad P\hat{O}C = C\hat{O}Q$

and $\quad P\hat{O}D = D\hat{O}R,$

$\therefore\ P\hat{O}C - P\hat{O}D = C\hat{O}Q - D\hat{O}R$

or $\quad D\hat{O}C = R\hat{O}Q - D\hat{O}C$

or $\quad R\hat{O}Q = 2D\hat{O}C.$

But $\quad D\hat{O}C = \text{angle between normals} = M\hat{O}X.$

$\therefore\ R\hat{O}Q = 2M\hat{O}X.$

Now $R\hat{O}Q$ is the angle between the two reflected rays.

Hence the reflected ray is rotated through twice the angle turned through by the mirror. This fact is made use of in the sextant, an instrument employed for measuring elevations.

Multiple Reflections.—Consider two mirrors MO and NO (fig. 19) placed so that the angle between their reflecting surfaces is one right angle. Let P be any point object between the mirrors, being, in general, not on the bisector of the angle MON.

An image of P is formed by reflection at mirror MO at a point Q, and another image of P is formed at R by reflection at mirror NO. Now Q is in front of NO (produced), and an image of Q is formed at I. Also at the point I is the image of R formed by reflection at mirror MO.

Since I lies behind both mirrors no further images are produced.

It can be seen that since PQ is bisected at right angles

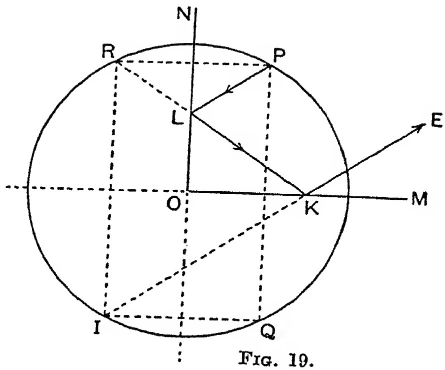


FIG. 19.

by MO, then OP=OQ. Similarly OP=OR and OQ=OI. Thus all the images lie on a circle which passes through the object and has its centre at the point of intersection of the mirrors.

In order to draw the path of the rays from the object to the eye when the eye views the final image, the position of the eye, E, is joined to the final image, I, cutting mirror MO at K. Now I is the image of R by reflection at mirror MO, so that the ray of light passes in the direction from R to K. Consequently R is joined to K, cutting NO at L. Here R is the image of the object, P, formed by reflection at NO so that the direction of the ray is from P to L. Thus the light passing from the object to the eye travels along the path PLKE. From the diagram it can be easily seen that at each reflection the angle of incidence equals the angle of reflection. If the position of the eye is changed so that the final image is viewed by reflection at mirror NO, then the path of the rays may be drawn in a similar manner.

In the example just considered, the angle between the

mirrors was 90°, and three images were formed. If the angle between the mirrors is 60° there are five images, for an angle of 45° seven images and for 30° eleven images.

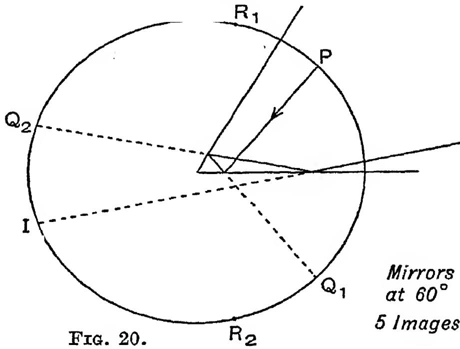


FIG. 20.

In general if the mirrors are inclined at an angle $\theta°$, the number of images formed is $\frac{360}{\theta}$, provided this is a whole number. This is obtained by considering that the circle may be split up into $\frac{360}{\theta}$ equal sectors, each of which contains one image, except the sector between the mirrors where only the object is situated, and the sector behind both mirrors, which contains two images. If $\frac{360}{\theta}$ is an even number, the two final images overlap and the number of images is then $\frac{360}{\theta} - 1$.

Two Parallel Mirrors.—A special case occurs when the mirrors are parallel with the reflecting surfaces facing each other. Here $\theta = 0$, and an infinite number of images is formed. These images all lie on a straight line passing through the object and cutting the mirrors at right angles. Then if M_1 and M_2 are the mirrors, I_1 is the image of O by reflection at M_1, I_{12} is the image of I_1 by reflection at M_2,

I_{121} the image of I_{12} by reflection at M_1, and so on. Another set of images—I_2, I_{21}, I_{212}—is also obtained, considering the first reflection to be at M_2. The path of the rays from

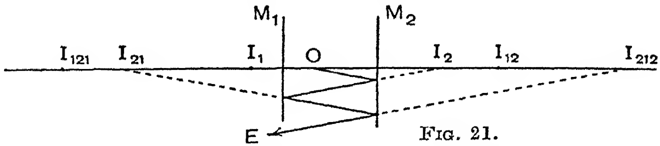


FIG. 21.

the object to the eye when the eye is viewing any image may be drawn as shown.

In the case of a mirror, the back of which is silvered, a series of images is formed in the same manner. Some of the light is reflected at the front face, and some by the silvered surface. Internal reflections also occur, so that a series of parallel rays—R_1, R_2, R_3—emerge from the

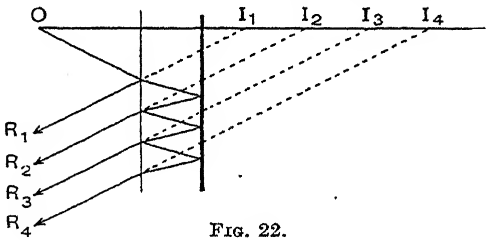


FIG. 22.

mirror. The intensities of the rays vary, R_2 always being much the brightest since most of the incident light passes into the glass of the mirror and is reflected by the silvered surface. The next brightest is generally R_1, and since absorption takes place in the glass, the other rays—R_3, R_4—have smaller and smaller intensities.

The Kaleidoscope consists of two or three plane mirrors contained in a cylindrical tube and inclined to each other generally at 60°. The mirrors give rise to symmetrical reflections of any objects placed at one end of the tube, so that by altering the positions of the objects numerous patterns may be obtained.

REFRACTION

When a ray of light passes from one transparent substance to another the direction of the ray in the second medium is generally not the same as in the first. This phenomenon is known as refraction, and the ray of light in the second medium is called the refracted ray. If a normal is drawn to the dividing surface at the point of incidence, then the angle between the normal and the incident ray is the angle of incidence, and the angle between the normal and the refracted ray is the angle of refraction.

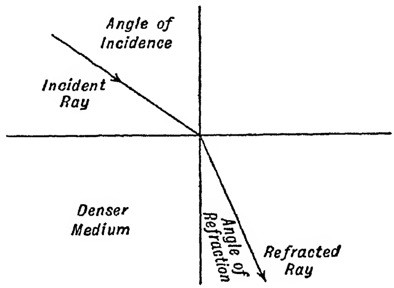


FIG. 23.

The laws of refraction are:

(1) The incident ray, the normal and the refracted ray are all in the same plane.

(2) The sine of the angle of incidence divided by the sine of the angle of refraction gives a constant, the value of which depends on the two media through which the light passes.

This second law was discovered by Snell in 1621 and consequently is often referred to as Snell's law. The ratio of the sine of the angle of incidence to the sine of the angle of refraction is called the refractive index of the second medium with regard to the first, and is denoted by the Greek letter μ.

The refractive index of a substance is usually measured when the ray of light passes from air—or more strictly a vacuum—to that substance.

These laws may be verified as follows:—

Place a parallel-sided slab of glass on a sheet of paper

and draw the outline of the slab ABCD. Put in two pins, K and L, upright on one side of the slab and observe these pins through the glass. Then place two more pins, M and N, in the paper on the opposite side of the slab to K and L so that on looking in the direction NM through the slab the four pins appear in line. Remove the slab and draw lines to join the positions of K and L, and M and N. Let KL produced meet AB in E, and NM produced meet CD in F. Then the path of the ray through the glass slab must be along EF. At E draw PQ normal to AB and measure the angles of incidence and refraction.

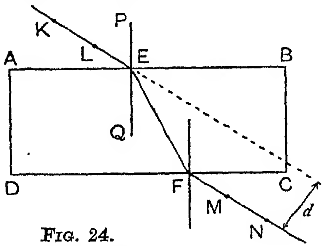


FIG. 24.

Instead of using pins, a line KE can be drawn on the paper and a straight edge placed so that it appears in line with KE seen through the slab. The straight edge FN then shows the position of the emergent ray.

If the experiment is repeated with different angles of incidence and the corresponding angles of refraction are measured, it is always found that sine $K\hat{E}P$ divided by sine $F\hat{E}Q$ gives the same result, thus verifying Snell's law. Also KL, PQ and EF are all in the plane of the paper, so that the first law of refraction is also verified. It will also be noticed that the emergent ray MN is parallel to the incident ray KL, so that the direction of the ray is not changed by refraction through a medium the opposite faces of which are parallel. The perpendicular distance, d, between the directions of the incident ray and the emergent ray is called the lateral displacement. This distance is zero when the angle of incidence is zero, and has its maximum value for the angle of incidence 90°.

Suppose in the above diagram the angles of incidence and refraction are denoted by i and r respectively. Let ${}_A\mu_G$ denote the refractive index of glass with respect to

air, and ${}_G\mu_A$ denote the refractive index of air with respect to glass.

Then ${}_A\mu_G=\frac{\sin i}{\sin r}$ at the first refraction, and ${}_G\mu_A=\frac{\sin r}{\sin i}$ at the second refraction, since the angle of emergence equals the angle of incidence. Thus ${}_A\mu_G=\frac{1}{{}_G\mu_A}$.

The experiment may be performed more easily by using a pencil of light obtained from a lamp and slits. The path of the ray may then be seen directly, and if a normal is drawn the angles of incidence and refraction can be measured.

Refraction through Several Media.—Suppose that a ray of light passes from air to a medium of refractive index μ_1, then into a different medium of refractive index μ_2 and finally emerges into air again. Let the angle of incidence be i and let α be the angle of refraction in the first medium. Then provided each substance has parallel sides, the angle of incidence from the first to the second medium will be equal to α. Let the angle of refraction in the second medium be β. The angle of emergence equals i. Then

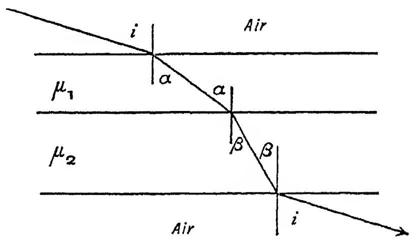


FIG. 25.

$$\mu_1=\frac{\sin i}{\sin \alpha}$$

and

$$_1\mu_2=\frac{\sin \alpha}{\sin \beta}.$$

Also

$$\frac{1}{\mu_2}=\frac{\sin \beta}{\sin i}.$$

Hence

$$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin \beta} \cdot \frac{\sin \alpha}{\sin i}$$
$$= \frac{\sin \alpha}{\sin \beta}$$
$$= {}_1\mu_2.$$

Thus the refractive index from a medium A to a medium B is the ratio of the refractive index of B to that of A.

For example, if A is water with refractive index 4/3, and B is glass with refractive index 3/2, then the refractive index of the glass with respect to water is $\frac{3/2}{4/3}$ or 9/8.

In the above cases the refractive index of air has been taken as unity. This is not quite accurate since the fundamental standard is the refractive index of a vacuum. With respect to a vacuum, air has a refractive index of 1·0003. Consequently if the refractive index of any substance with regard to air is μ, the true refractive index of that substance is 1·0003 μ. In general, the factor 1·0003 makes such a small difference that it is neglected and the value of the refractive index taken with respect to air at the atmospheric pressure prevailing.

Formation of an Image by Refraction.—Let O be a point object on the base of a slab of glass which has parallel sides.

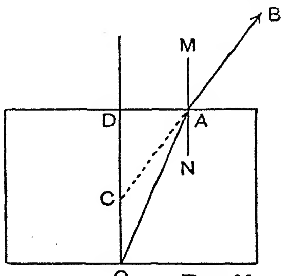


FIG. 26.

Then any ray OA from O will pass through the glass and be refracted out along AB. To the eye the object O appears

to be somewhere along the direction BA. Let BA produced meet the vertical OD through O at the point C, and let MN be the normal through A.

Then

$$\mu=\frac{\sin B\hat{A}M}{\sin O\hat{A}N}=\frac{\sin A\hat{C}D}{\sin A\hat{O}D}$$

$$=\frac{AD/AC}{AD/AO}=AO/AC.$$

Now if $A\hat{O}D$ is a small angle, AD is small and

$$\frac{AO}{AC}=\frac{DO}{DC}.$$

This is often expressed as

$$\mu=\frac{\text{real depth}}{\text{apparent depth}},$$

where DO is the depth or thickness of the slab and DC is the apparent thickness. Now the narrow pencil of light received by an eye vertically above O consists of rays which make small angles with the normals, and consequently to the eye in such a position the rays appear to diverge from C, so that C is the virtual image of O.

If the eye is not vertically above the object then the

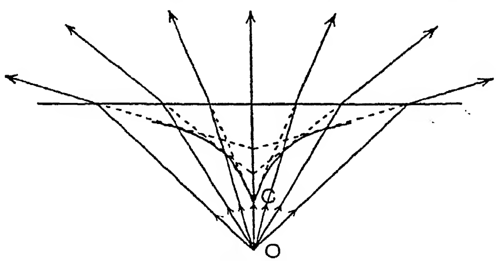


Fig. 27.

position of C changes. As $A\hat{O}D$ gradually increases, C moves along OD towards D, so that the apparent depth

gets less and less. This shows why a pool of water seems to be shallower the farther we are from it. The directions of the rays after refraction out of the slab touch a curve which is known as a *caustic*. This curve has a cusp at C (fig. 27), where C is the position of the image of O. This affords a simple method for the determination of μ for such substances as glass. A pin is placed in contact with one face of a slab of the material and two more pins are placed on the opposite side of the slab so that the three pins appear to be in line when the first pin is viewed through the slab. By joining the positions of the last two pins the direction of the refracted ray is found. Several more positions are taken, the first pin remaining undisturbed. Finally the slab is removed, the caustic curve drawn and the position of the image of the first pin obtained. From this the apparent thickness of the slab can be found, and since the real thickness may be measured, μ can be calculated.

This method is not a very accurate one. A much better result is obtained by using a travelling microscope which may be raised or lowered vertically. The microscope is first focussed on a pencil mark made on a piece of paper. The slab of the refracting material is then placed over the paper and the microscope focussed on the mark as seen through the slab. Finally the microscope is focussed on the top surface of the slab—usually there is sufficient dust on this surface to focus on. Then the difference between the first and last readings of the microscope gives the real depth, while the difference between the second and third readings gives the apparent depth.

Total Internal Reflection.—When a ray of light passes from one medium to an optically denser medium (*e.g.* air to glass) the ray is bent towards the normal. If the ray is reversed so that it passes into the rarer medium, then it is bent away from the normal.

Thus if O is an object under water and AB is the boundary surface between the water and air, the paths of the rays from O are as shown by OCG. If a is the angle between

the normal and the ray in water while β is the corresponding angle for the ray in air, then

$$\mu \sin \alpha = \sin \beta.$$

As α increases, a value is reached for which

$$\mu \sin \alpha = 1 \quad \text{or} \quad \beta = 90°.$$

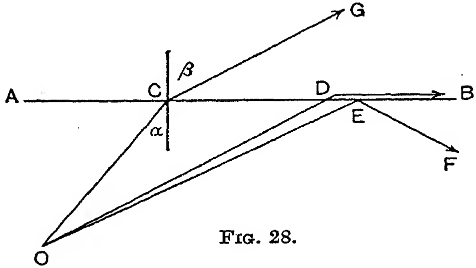


FIG. 28.

This is indicated by the ray ODB.

No further increase in the value of $\mu \sin \alpha$ is possible since $\sin \beta$ can never be greater than unity. This limiting value is thus given by $\sin \alpha = 1/\mu$. For all angles greater than this there can be no refraction, and so the whole of the light is reflected as indicated by the ray OEF. This effect is thus known as *total internal reflection*, and the value of α for which $\sin \beta$ equals unity is called the *critical angle*. It will thus be observed that the critical angle has a value which depends on the refractive index, since critical angle $= \sin^{-1}\left(\frac{1}{\mu}\right)$

For crown glass the critical angle is approximately 42°, and for water its value is nearly 49°. For light passing from glass to water the ratio of the refractive indices is 9/8, so that the critical angle is $\sin^{-1}(\frac{8}{9})$ or nearly 63°.

Simple instances of total internal reflection are as follows:—

(1) Place an empty test-tube in a beaker of water which is illuminated from one side. The tube presents a silvery

appearance since most of the light falling on it is reflected at the boundary between the glass of the tube and the air. On filling the tube with water the polished appearance vanishes, since the value for the critical angle is increased and very little of the light is reflected.

(2) Shine a beam of light into one end of a glass rod which is bent in the form of an arc. The whole of the light emerges from the opposite end, none passing out through the sides of the glass rod. The reason for this is that the light meets the sides of the rod at angles greater than the critical angle, and so is all reflected.

Totally Reflecting Prisms.—A prism is defined as a body bounded by three planes intersecting in three parallel straight lines. The prism is a right-angled isosceles one when the angle between two of the planes is 90° and the angles between each of these planes and the third plane are 45°. If a ray of light falls normally on the side AB of such a prism it passes through undeviated and strikes the hypotenuse face at an angle of incidence 45°.

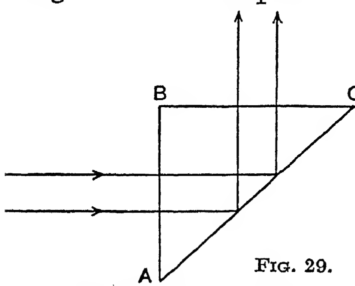


FIG. 29.

Now if the prism is of glass, this angle is greater than the critical angle, and the ray is totally reflected. The reflected ray passes out normally to BC, so that the ray is deviated through 90° by the reflection. If the incident light falls normally on the hypotenuse face then it passes into the prism and is reflected at the face AB and again at BC, so that the ray is turned through 180°.

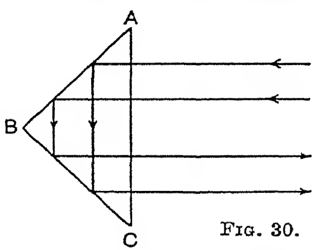


FIG. 30.

This prism may also be used to invert a beam of light. If an inverted image is formed by any apparatus (*e.g.* an

optical lantern), then the image can easily be made erect by passing the light through a prism in the manner indicated. The light is refracted at the face BC and meets

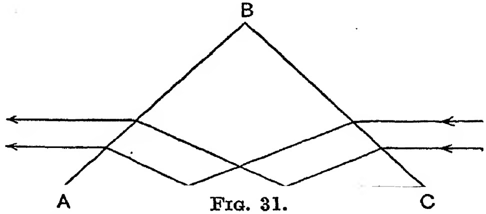


FIG. 31.

the hypotenuse face at an angle of incidence greater than the critical angle. Reflection occurs, and the light is then refracted out through AB so that no deviation has occurred, but inversion has taken place.

A prism such as described is called a totally reflecting prism, although when used as in the last example it may be called an erecting prism. It is often used in preference to a mirror when it is required to reflect a beam of light through 90° or 180°, since there is only a very small loss of intensity of light.

Refraction in Nature.—It has already been said that refraction occurs when light passes from one medium to another, the optical densities of the media being different. In the case of gases, fairly small changes of temperature are sufficient to cause variations in the density, so that the refractive index varies and a noticeable deviation in a ray of light occurs. If an object is observed over the top of a Bunsen flame the image seen is not clearly defined. This is due to fluctuations in the temperature causing slight changes in the refractive index of the air in that neighbourhood, so that the directions of the rays reaching the eye vary.

The density of the air decreases the more distant we are from the surface of the earth. A corresponding decrease in the refractive index of the air takes place at the same time, so that light from the sun (or stars) enters the eye

in a direction slightly different from that with which it left the sun. This effect is greater the less the elevation of the sun, and at sunrise or sunset light from the sun reaches the eye although at that time the sun is actually below the plane of the horizon.

In the deserts the air nearest the sand is at the highest temperature, and at higher levels the air becomes gradually cooler. The refractive index of the air thus varies for different levels, having its minimum value near to the surface of the sand. Light from the sky passes into layers of air which become successively less dense, and total reflection may occur. To an observer who receives the rays, the sky appears below the surface of the sand. This produces the illusion that a pool of water is in the distance, the reflection occurring at the surface of the water. This phenomenon is known as a mirage. A similar effect, but with refractive indices in the reverse order, can sometimes be seen over the surface of the sea when an image of a ship appears inverted in the sky.

Simple Methods for the Determination of Refractive Indices.—1. *Measurement of the Angles of Incidence and Refraction.*—This method has already been described in the verification of Snell's law. The sines of the angles of incidence and refraction are compared and the refractive index found from $\mu = \frac{\sin i}{\sin r}$.

2. *Measurement of the Real and Apparent Depth.*—Any transparent material always appears to be less thick than it actually is when an object is viewed through it, and it has been shown that $\mu = \frac{\text{real depth}}{\text{apparent depth}}$. This method of finding a value for μ has already been explained in the case of solids, but it is equally easy to carry out for liquids. A small bright object O is placed at the bottom of a tank containing the liquid, and a second object—*e.g.* a knitting needle N—is supported above the surface of the liquid. When viewed from above, a reflection of the needle in the

surface of the liquid is seen, and this image I is made to coincide with the apparent position of the object O. Then

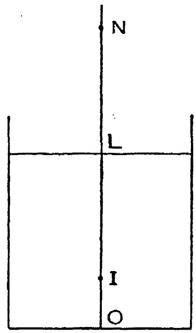


FIG. 32.

the image of N is as far below the surface of the liquid as N is above, *i.e.* NL = LI. Hence NL equals the apparent depth, and the real depth, LO, is found by direct measurement, so that μ may be determined. Observations are made for different depths of the liquid and several readings for μ taken.

3. *Displacement Method.*—When a ray of light passes through a slab of glass the opposite sides of which are parallel, the direction of the ray is unchanged, but the ray

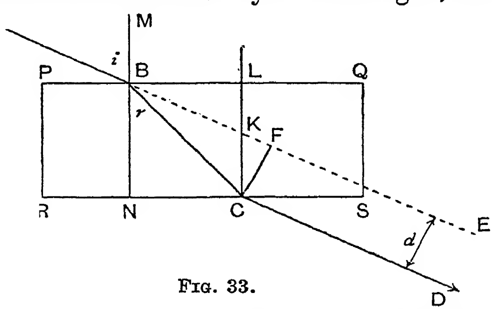


FIG. 33.

itself is laterally displaced. This lateral displacement depends on the angle of incidence, and if both these quantities

are measured μ may be calculated. Let ABCD be the path of the ray, ABE being the original path before the glass slab PQRS is placed in position. The lateral displacement, d, equals CF. If MN and LC are the normals at B and C respectively, and ABE cuts LC in K,

$$\begin{aligned} d = \mathrm{CF} &= \mathrm{CK} \sin \mathrm{C\hat{K}F} \\ &= \mathrm{CK} \sin i \quad \text{since} \quad \mathrm{C\hat{K}F} = \mathrm{A\hat{B}M} \\ &= (\mathrm{LC} - \mathrm{LK}) \sin i \\ &= (t - \mathrm{LB} \cot i) \sin i, \end{aligned}$$

where

$$t = \mathrm{LC} = \text{thickness of slab.}$$

But

$$\mathrm{LB} = \mathrm{NC} = \mathrm{NB} \tan r = t \tan r.$$

Hence

$$d = t\left(1 - \frac{\tan r}{\tan i}\right) \sin i.$$

From

$$\mu = \frac{\sin i}{\sin r}$$

we have

$$\tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$$

and thus

$$d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2}} \right. \quad \sin i.$$

To carry out the experiment draw four or five lines parallel at a distance of five millimetres apart. The end one should be produced so that when the slab is placed on the paper this line extends on both sides of it. The slab is then turned so that the second line, viewed through the slab, appears to be in line with the continuation of the first one. To secure exact alignment pins may be placed vertically along the lines. The angle of incidence is then measured, and the experiment is repeated so that the third and

then the fourth and fifth lines appear, in turn, to be continuations of the first line. The lateral displacement

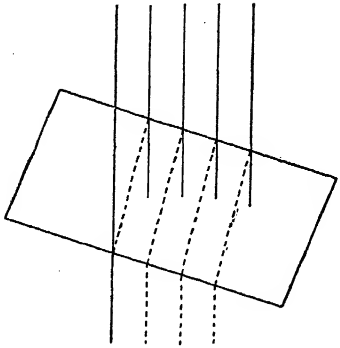


FIG. 34.

is known in each case, so that a value for μ may be obtained.

4. *By Measurement of the Critical Angle.*—The substance is taken in the form of a slab, and a pin is placed touching the slab and about one centimetre from one of the corners. The pin, A, is viewed through the adjacent side of the slab, and two more pins, B and C, are placed in to mark the direction of vision for which A just seems to disappear. The slab is outlined and then removed, and the path of the rays CBDA is drawn. Then the angle between DA and the normal at A must be the critical angle since no ray from A entered the slab in the direction AD. A value for μ is then found from $\mu = \frac{1}{\sin c}$, where c is the critical angle.

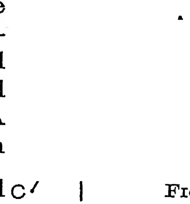


FIG. 35.

This method is unsuitable if $c < 45°$, for if $c < 45°$ then $A\hat{D}E > 45°$, or $A\hat{D}E > c$, and no ray such as DB emerges.

This method may also be used for determination of the

refractive indices of liquids, provided the slab has a known refractive index. Instead of the pin A, a thin strip of paper previously dipped into the liquid is placed against the side of the slab and the position for which this strip seems to vanish is marked. As before, the critical angle, ϕ, is measured. This is the critical angle between the liquid and the material of the slab since there is a thin layer of liquid between the strip of paper and the slab. Denoting the refractive index of the liquid by ${}_A\mu_L$ and of the material of the slab by ${}_A\mu_G$, both with respect to air, the refractive index of the slab with respect to the liquid being ${}_L\mu_G$, then

$$ {}_A\mu_G = {}_A\mu_L \cdot {}_L\mu_G $$

or

$$ {}_L\mu_G = \frac{{}_A\mu_G}{{}_A\mu_L}. $$

Now ${}_L\mu_G = \frac{1}{\sin\phi}$ and is thus known. Since ${}_A\mu_G$ is also known, or may be found, a value for ${}_A\mu_L$ may be obtained.

5. *By Aid of a Concave Mirror.*—A concave mirror is placed on the table and an object (a pin or an illuminated aperture) is held above this until the object and image coincide. The distance of the object from the mirror is then measured (PC). A little of the liquid whose refractive index is required is then placed in the mirror so that it forms a plano-convex lens. Again coincidence of object and image is obtained, the reflection again taking place at the mirror. The distance of the object from the mirror is once more measured (PO).

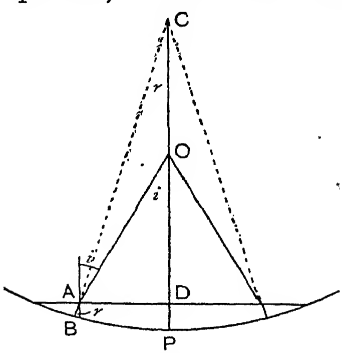


Fig. 36.

Then for the liquid, $\mu = \frac{PC}{PO}$.

Taking i and r as the angles of incidence and refraction at the boundary between air and liquid, it is seen that $A\hat{O}P = i$ and $A\hat{C}P = r$.

Hence

$$\mu = \frac{\sin i}{\sin r} = \frac{AD/AO}{AD/AC} = \frac{AC}{AO},$$

which for small angles of incidence and refraction (as obtained in practice) becomes $\frac{PC}{PO}$, or $\mu = \frac{PC}{PO}$.

Refractive Index of Crystals.—It is sometimes important in the study of crystallography to know the refractive index of a crystal. This may be determined very simply in the case of colourless crystals. If such a crystal is placed in a colourless liquid of the same refractive index its boundaries disappear owing to the refractive index throughout being constant. By employing a series of liquids of different refractive indices it is thus possible to find the refractive index of a crystal between quite narrow limits. These limits between which the refractive index lies can be made much closer by employing the fact that the refractive index of a liquid depends on its temperature and on the wave-length of the light. By varying these factors several readings can be made and an accurate value for the refractive index of the crystal obtained.

EXAMPLES ON CHAPTER II

1. ABCD is a rhombus, the angle ABC being 60°. AB, BC are plane mirrors, and a small object is placed at the middle point of CD. Show by means of diagrams the paths of the rays by which the various images of the object are seen by an eye placed at the middle point of DA. (Camb. Schol.)

2. A point object is placed between two plane mirrors which are inclined to each other at 72°. Draw a diagram to show the positions of the images formed.

3. An object is placed between two parallel mirrors, which are 3 cm. apart, so that it is 2 cm. from one of the mirrors. Find the distance between the third image formed by each mirror. Draw the rays to an eye situated between the mirrors when the eye is viewing one of these images.

4. A ray which is incident on a mirror passes through a point X. The reflected ray passes through a point Y. Show that the ray travels from X to Y by the shortest path when the angles of incidence and reflection are equal.

5. State the laws of refraction of light. The cross-section of a glass prism is a triangle ABC. AB and BC are at right angles to one another, and CA makes an angle of 45° with the other two sides. A ray of light enters the prism by passing normally through the face AB. If total internal reflection is to take place at the face AC, what is the least value of the refractive index of the glass? (O. & C.)

6. A pin is held horizontally above a tank of water so that the image by reflection of the pin appears to coincide with an object at the bottom of the tank. More water is poured into the tank so that the depth is increased by 4 inches. The pin now has to be raised 7 inches for coincidence to occur again. Find the refractive index of water.

7. Determine the lateral displacement of a ray of light which is incident at 45° on a slab of glass of thickness 5 cm. Take μ as 1·5.

8. State the laws of refraction, and describe how you would verify them experimentally.

9. A ray of light is incident on a glass prism of angle 30° so that reflection occurs at the second face of the prism, and the ray travels back along the same path. Show that the angle of incidence is $\sin^{-1}\left(\frac{\mu}{2}\right)$.

Prove that it is impossible for light to travel directly through a prism without reflection if the angle of the prism is greater than $2\sin^{-1}\left(\frac{1}{\mu}\right)$.

10. Calculate the apparent displacement of a small object produced by interposing a transparent plate of thickness t and refractive index μ between object and observer.

A microscope is focussed on a scratch on the upper surface of a transparent plate of thickness 1 mm. In order to focus on the image

of the scratch reflected from the lower surface of the plate the microscope must be screwed down through a distance 1·25 mm. Calculate the refractive index of the material of the plate.

(Lond. H.S.C.)

11. Describe what happens when a ray of light passes into a less dense medium as the angle of incidence at the boundary face increases.

Show that to a fish in water of refractive index 4/3 all objects outside the water appear to lie within a cone which has a vertical axis, the semi-vertical angle being $\sin^{-1}(\frac{3}{4})$.

12. Define the terms: critical angle, refractive index, and show how they are related to each other.

Calculate the critical angle for a ray of light passing from a medium of refractive index 3/2 into a medium of refractive index 4/3.

(Lond. Inter.)

13. Explain the meaning of critical angle, and describe how you would measure the critical angle for a water air boundary.

ABCD is the plan of a glass cube. A horizontal beam of light enters the face AB at grazing incidence. Show that the angle θ, which any rays emerging from BC would make with the normal to BC, is given by $\sin\theta = \cot C$, where C is the critical angle. What is the greatest value that the refractive index of glass may have if any of the light is to emerge from BC? (N.)

14. Explain the meaning of critical angle and total internal reflection. Describe briefly (*a*) one natural phenomenon due to total internal reflection, (*b*) one practical application of it.

Light from a luminous point on the lower face of a rectangular glass slab 2·0 cm. thick strikes the upper face, and the totally reflected rays outline a circle of 3·2 cm. radius on the lower face. What is the refractive index of the glass? (N.)

15. If a candle flame is viewed very obliquely in a plate glass mirror a series of images of the flame is seen, the second of which is much the brightest.

If an iron ball is covered with soot and held under water it presents the appearance of polished silver.

Explain these observations. (O. & C.)

16. Give an account of the laws of refraction of light.

A slab of glass with parallel sides is wholly immersed in water. Trace the path of a beam of light through the glass slab when the beam makes an angle of 30° with the normal to one of the faces.

(Refractive index of glass 1·5, of water 1·33.) (O. & C.)

17. A pin is held above a concave mirror, which is placed horizontally, and is moved up or down until it coincides with its image. It is then at a distance x from the mirror. Water is poured into the mirror, and now the pin has to be moved a distance y nearer to the mirror for it to coincide with its image. Show that the refractive index of water is $\frac{x}{x-y}$.

18. Describe a method of finding the refractive index of a liquid, of which only a few drops are supplied.

19. State the laws of refraction of light and show that light can pass from a dense to a rare medium only within a limiting angle of incidence. (Lond. H.S.C., Subsidiary.)

20. Show that if a horizontal concave mirror is filled with a liquid its apparent radius of curvature is diminished in the ratio of the refractive index of the liquid. (Lond. Inter.)

21. Two parallel-sided plates of glass are cemented together so as to enclose a thin film of air. Show that, if the system is immersed in water, a ray of light will cease to be transmitted when its angle of incidence on the first glass surface is greater than the critical angle between water and air.

How can the principle of total reflection be employed to determine the refractive index of a solid? (O. & C.)

CHAPTER III

SPHERICAL MIRRORS AND THIN LENSES

The Sign Convention.—At the present time there is a certain amount of confusion as to which is the best sign convention to apply to mirrors and lenses. Three conventions are much more widely used than the others and therefore all formulæ in this book are obtained by each of these three conventions. **It must be emphasised that it is only necessary to know, and understand, one of these conventions.** The three conventions are:

(A) *Distances are measured from the mirror or lens and are counted positive when measured in a direction opposite to that in which the incident light travels. On this convention concave mirrors and lenses have positive focal lengths while convex mirrors and lenses have negative focal lengths.*

(B) *Distances are measured from the mirror or lens and are counted positive when measured in the same direction as that in which the incident light travels. On this convention concave mirrors and lenses have negative focal lengths, while convex mirrors and lenses have positive focal lengths.*

(C) *Distances are measured from the mirror or lens and are counted positive if actually traversed by the light, but negative if the light only appears to travel in that direction. Thus for a real image v is positive, and for a virtual image v is negative. Also concave mirrors and convex lenses have positive focal lengths, while convex mirrors and concave lenses have negative focal lengths.*

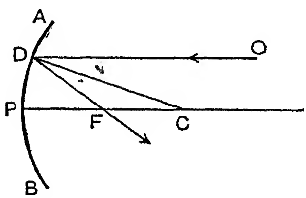
Spherical Mirrors.—A spherical mirror is a reflecting surface which forms part of a sphere. The mirror may be either convex or concave. For the former the centre of curvature is behind the mirror, while for the latter

it is in front. Thus a shaving mirror is concave while a motor-car mirror is convex.

In the following theory any refraction owing to the thickness of the glass of the mirror is neglected. The theory is accurate for black glass mirrors or for mirrors silvered on their front surfaces.

In the following constructions for concave and convex mirrors let APB represent the mirror, P being the pole of the mirror, and let C be the centre of curvature. The centre of curvature is the centre of the sphere of which the mirror is a portion, and the pole of the mirror is the centre of the mirror itself. The line passing through P and C is called the optic or principal axis of the mirror. The radius of curvature equals PC. The point F through which all the rays from infinity pass, or appear to pass, after reflection is known as the focus of the mirror and the length PF is called the focal length.

It can be shown that the focal length of a mirror is half the radius of curvature. Let OD be a ray from infinity meeting the mirror at D and cutting PC in F after reflection.



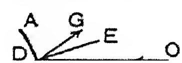


FIG. 37.

If CD is the normal at D, then for the concave mirror

$$O\hat{D}C = C\hat{D}F.$$

But

$$O\hat{D}C = D\hat{C}F \quad \text{since} \quad OD \text{ is parallel to } PC.$$

Hence

$$C\hat{D}F = D\hat{C}F.$$

$$\therefore\ DF = FC.$$

Now

$$DF = PF \text{ (approximately)}$$

so that $$\mathrm{PF} = \mathrm{FC} \text{ (approximately)}.$$
For the convex mirror
$$\mathrm{O\hat{D}E} = \mathrm{E\hat{D}G},$$
where DG is the reflected ray, which when produced backwards passes through F.
But
$$\mathrm{O\hat{D}E} = \mathrm{D\hat{C}F}$$
and
$$\mathrm{E\hat{D}G} = \mathrm{C\hat{D}F} \text{ (vertically opposite).}$$
Hence
$$\mathrm{D\hat{C}F} = \mathrm{C\hat{D}F}.$$
$$\therefore \ \mathrm{DF} = \mathrm{FC},$$
giving as before
$$\mathrm{PF} = \mathrm{FC}.$$
Thus F lies midway between P and C.

General Equation for Spherical Mirrors.—Two cases will now be considered for concave mirrors, firstly the object O lying beyond F, and secondly, O lying between F and the mirror.

(1) Let the object be a point source of light on the optic axis, and let the incident ray OD make an angle α with

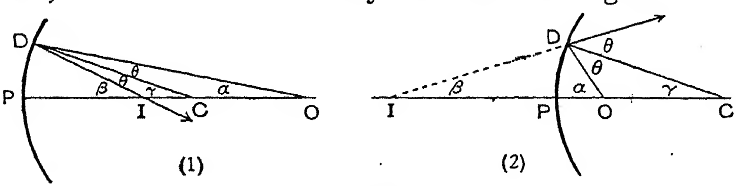


FIG. 38.

PC, and θ with CD, the normal. Then the reflected ray DI also makes an angle θ with CD and meets PC in I. Call $\mathrm{D\hat{I}P}$ β and $\mathrm{D\hat{C}P}$ γ, and let the distances PO, PC and PI be denoted by u, r, v respectively.

Then
$$\beta = \theta + \gamma \quad \text{and} \quad \gamma = \theta + \alpha.$$
Eliminating θ we have
$$\alpha + \beta = 2\gamma.$$

USING CONVENTION **A**	USING CONVENTION **B**	USING CONVENTION **C**
u, v, and r are positive	u, v, and r are negative	u, v, and r are positive
$\therefore \alpha=\frac{\mathrm{PD}}{u}$, $\beta=\frac{\mathrm{PD}}{v}$, $\gamma=\frac{\mathrm{PD}}{r}$,	$\therefore \alpha=-\frac{\mathrm{PD}}{u}$, $\beta=-\frac{\mathrm{PD}}{v}$, $\gamma=-\frac{\mathrm{PD}}{r}$,	$\therefore \alpha=\frac{\mathrm{PD}}{u}$, $\beta=\frac{\mathrm{PD}}{v}$, $\gamma=\frac{\mathrm{PD}}{r}$,

since α, β, and γ are small angles. Hence

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}.$$

It is obvious from the diagram that the object and image are interchangeable.

(2) In the second case the reflected ray produced back meets CP to the left of P, and the equations now are

$$\alpha=\theta+\gamma \quad \text{and} \quad \theta=\beta+\gamma,$$

giving $$\alpha-\beta=2\gamma.$$

In this case

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\alpha=\frac{\mathrm{PD}}{u}$, $\beta=-\frac{\mathrm{PD}}{v}$, $\gamma=\frac{\mathrm{PD}}{r}$,	$\alpha=-\frac{\mathrm{PD}}{u}$, $\beta=\frac{\mathrm{PD}}{v}$, $\gamma=-\frac{\mathrm{PD}}{r}$,	$\alpha=\frac{\mathrm{PD}}{u}$, $\beta=-\frac{\mathrm{PD}}{v}$, $\gamma=\frac{\mathrm{PD}}{r}$,

and the final equation again is $1/u+1/v=2/r$.

For a convex mirror the image is always behind the

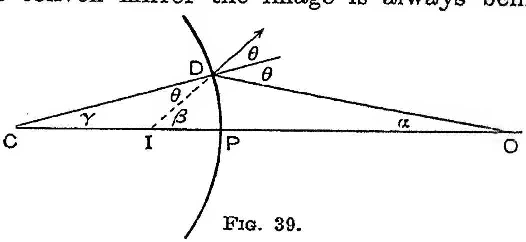


FIG. 39.

mirror irrespective of the position of the object. The equations then are (fig. 39)

$$\alpha+\gamma=\theta \quad \text{and} \quad \gamma+\theta=\beta,$$

whence $$\beta - \alpha = 2\gamma.$$

Since

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\alpha = \frac{\text{PD}}{u}, \quad \beta = -\frac{\text{PD}}{v},$	$\alpha = -\frac{\text{PD}}{u}, \quad \beta = \frac{\text{PD}}{v},$	$\alpha = \frac{\text{PD}}{u}, \quad \beta = -\frac{\text{PD}}{v},$
$\gamma = -\frac{\text{PD}}{r},$	$\gamma = \frac{\text{PD}}{r},$	$\gamma = -\frac{\text{PD}}{r},$

we obtain $$\frac{1}{u} + \frac{1}{v} = \frac{2}{r},$$

the general formula for mirrors.

If the object is at infinity, $1/u$ equals zero and consequently v equals $r/2$. But in this case the reflected rays pass through the focus, and the distance v is the focal length, usually denoted by f. Thus we have, as before,

$$f = \tfrac{1}{2}r.$$

Graphical Construction.—So far the object has been considered as a point on the optic axis. If the object is a point not on the axis then the image is a point which also does not lie on the axis. The position of the image

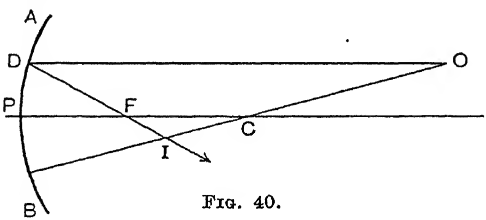


FIG. 40.

is found conveniently by a graphical method. One ray from the object at O is taken parallel to the axis so that after reflection at the mirror AB it passes through the focus F. Thus OD is the incident ray, and DF the reflected ray. Another ray from O is considered, this one passing through C, the centre of curvature. After reflection at the mirror this ray is reflected back along the same path

4

since it strikes the mirror normally. Then the image of O lies at I, where the two reflected rays meet.

If the object is not a point, but has a length OK perpendicular to the axis, then the position of the image of K may be determined by drawing rays as mentioned above. Then if similar rays from any other point X on OK are

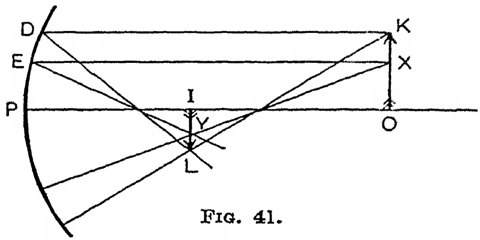


FIG. 41.

taken the consequent reflected rays must meet at some point Y on IL, where L is the image of K and IL is drawn at right angles to the axis. This follows since

$$\frac{XK}{YL}=\frac{OK}{IL}=\frac{PD}{IL}=\frac{DE}{YL},$$

neglecting the curvature of the mirror.

Since XK=DE, this shows that the two reflected rays pass through the same point Y on IL. So any point on

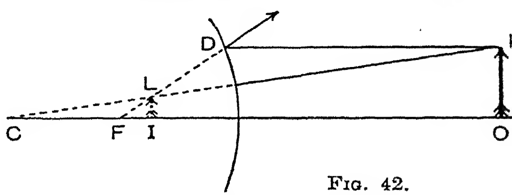


FIG. 42.

OK has a point image on IL, and consequently IL is the image of OK.

The construction is similar in the case of convex mirrors. If OK is the object (fig. 42), IL is the image. This image is always situated between the pole and F whatever the position of the object.

By this method we may also obtain the general formula for mirrors.

The length DP is very small compared with PC, and may in practice be considered as straight and perpendicular

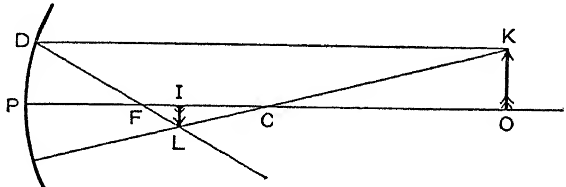


FIG. 43.

to PC. Then we have two sets of similar triangles, KCO and CIL, DFP and FIL. Hence

$$\frac{OK}{IL}=\frac{CO}{CI} \quad \text{and} \quad \frac{PD}{IL}=\frac{FP}{FI}.$$

But OK = PD and so

$$\frac{CO}{CI}=\frac{FP}{FI}$$

or

$$\frac{u-2f}{2f-v}=\frac{f}{v-f}$$

or

$$uv-uf-2vf+2f^2=2f^2-fv$$

which gives

$$uv=fu+fv.$$

Dividing by uvf,

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

or

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}.$$

Magnification.—The ratio of the length of the image to that of the object, *i.e.* $\frac{IL}{OK}$, is called the linear magnification,

denoted by m. If we draw a ray from K to P, then the

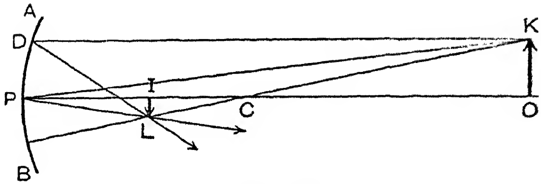


FIG. 44.

reflected ray passes through L and the magnification is given by

$$m = \frac{IL}{OK} = -\frac{PI}{PO} = -\frac{v}{u},$$

since the triangles KOP and ILP are similar.

The minus sign indicates that if u and v have the same sign then the image is inverted, but that if u and v are opposite in sign the image is erect.

We may thus classify our results for the position and nature of the image for different positions of the object.

Position of object	Position of image	Nature of image
Concave Mirrors		
At infinity	At F	Real
Between ∞ and C	Between F and C	Real, inverted, diminished
At C	At C	Real, inverted, same size
Between C and F	Between C and ∞	Real, inverted, magnified
At F	At infinity	
Between F and P	Between ∞ and P	Virtual, erect, magnified
At P	At P	Erect, same size
Convex Mirrors		
At infinity	At F	Virtual
Between ∞ and P	Between F and P	Virtual, erect, diminished
At P	At P	Erect, same size

Refraction at Spherical Surfaces.—We must first consider what signs are to be taken for the radii of curvature of the surfaces separating the two media. Using Convention:

(A) Concave surfaces have positive radii of curvature and convex surfaces have negative radii of curvature.

(B) Concave surfaces have negative radii of curvature and convex surfaces have positive radii of curvature.

(C) The radius is positive if the refraction at this surface increases the convergency (or decreases the divergency) of a beam of light entering the medium from air. Hence a concave surface has a negative radius of curvature and a convex surface has a positive radius of curvature.

Let the boundary between two transparent media be spherical in shape, the centre of curvature being at C. Consider the surface to be a concave one with respect to a point object O on the optic axis. Let the refractive indices be μ_1 and μ_2, and denote the ratio $\frac{\mu_2}{\mu_1}$ by μ, assuming that μ is greater than unity.

Join OC and let OC produced cut the surface at P. Let

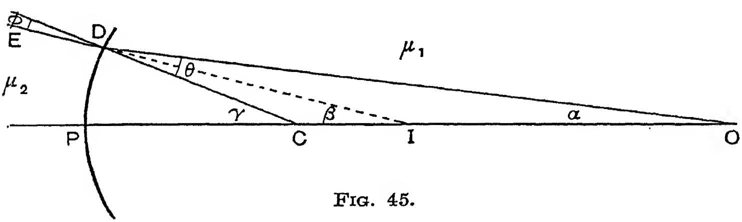


FIG. 45.

OD be a ray making a small angle α with OC, and let CD be the normal at D. After refraction the ray moves in a direction DE as though it came from a point I on OP; let it make an angle ϕ with CD produced. I is thus the virtual image of O. Also let $O\hat{D}C$ be θ, $D\hat{I}P$ be β, $D\hat{C}P$ be γ. Then

$$\mu_1 \sin \theta = \mu_2 \sin \phi.$$

But since α is small, θ and ϕ are also small, and we may

write
$$\mu_1\theta=\mu_2\phi$$
or
$$\theta=\mu\phi, \qquad (\mu=\mu_2/\mu_1).$$
Now
$$\theta+\alpha=\gamma \quad \text{and} \quad \gamma-\beta=\phi.$$
Substituting for θ and ϕ we get
$$(\gamma-\alpha)=\mu(\gamma-\beta)$$
or
$$\mu\beta-\alpha=(\mu-1)\gamma.$$

CONVENTION **A**

$$\alpha=\frac{\text{PD}}{u}, \quad \beta=\frac{\text{PD}}{v}, \quad \gamma=\frac{\text{PD}}{r},$$
$$\therefore \frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$$
or
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}.$$

CONVENTION **B**

$$\alpha=-\frac{\text{PD}}{u}, \quad \beta=-\frac{\text{PD}}{v}, \quad \gamma=-\frac{\text{PD}}{r},$$
$$\therefore \frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}$$
or
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}.$$

CONVENTION **C**

$$\alpha=\frac{\text{PD}}{u}, \quad \beta=-\frac{\text{PD}}{v}, \quad \gamma=-\frac{\text{PD}}{r},$$
$$\therefore \frac{\mu}{v}+\frac{1}{u}=\frac{\mu-1}{r}$$
or
$$\frac{\mu_2}{v}+\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$$

The treatment is the same as above for all positions of O, provided the surface is a concave one. For convex surfaces, however, two cases have to be considered, one where I falls to the left of the surface and is a real image, the other where I is on the same side as the object and is virtual.

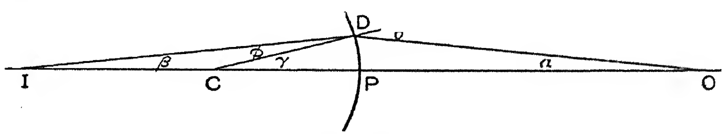


FIG. 46.

In the former case (fig. 46)
$$\phi=\gamma-\beta \quad \text{and} \quad \theta=\gamma+\alpha,$$
giving
$$\gamma+\alpha=\mu(\gamma-\beta)$$
or
$$\alpha+\mu\beta=(\mu-1)\gamma.$$

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\alpha = \frac{PD}{u} \quad \beta = -\frac{PD}{v}$	$\alpha = -\frac{PD}{u}, \quad \beta = \frac{PD}{v},$	$\alpha = \frac{PD}{u}, \quad \beta = \frac{PD}{v},$
$\gamma = -\,PD$	$\gamma = \frac{PD}{r},$	$\gamma = \frac{PD}{r},$
$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$	$\frac{\mu}{v} - \frac{1}{u} : \quad \mu - 1$	$\therefore \frac{\mu}{v} + \frac{1}{u} = \frac{\mu - 1}{r}.$

For the second case (fig. 47) the equations are

$$\phi = \beta + \gamma \quad \text{and} \quad \theta = \gamma + \alpha,$$

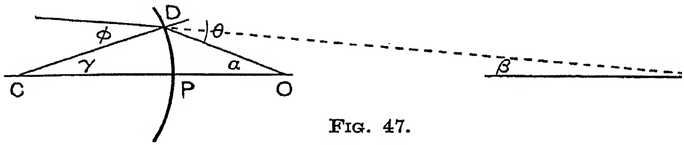


FIG. 47.

giving $\gamma + \alpha = \mu(\beta + \gamma)$

or $\mu\beta - \alpha = \gamma(1 - \mu).$

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\alpha = \frac{PD}{u}, \quad \beta = \frac{PD}{v},$	$-\frac{PD}{u}, \quad \beta = -\frac{PD}{v},$	$\frac{PD}{u}, \quad \beta = -\,PD$
$\gamma = -\frac{PD}{r},$	$\gamma = \frac{PD}{r},$	$\gamma = \frac{PD}{r},$
$\therefore \frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}.$	$\frac{1}{u} \quad \mu - 1.$	$\frac{\mu}{v} + \frac{1}{u} : \quad \mu - 1.$

Magnification Produced by Spherical Refracting Surfaces.—If the object is of finite size an image of finite size is formed, and its position may be found by a graphical method.

An object OK gives rise to an image IL. Two incident

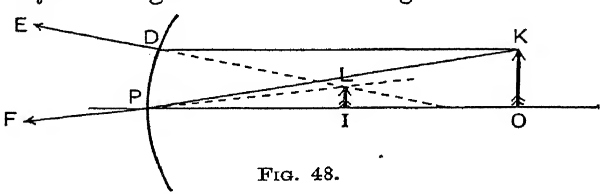


FIG. 48.

rays KD and KP are taken, and by using $\mu = \sin i / \sin r$ the refracted rays DE and PF are drawn. These rays meet

at L when produced back, so that L is the image of K and IL the image of OK.

Now
$$\mu=\frac{\sin \mathrm{K\hat{P}O}}{\sin \mathrm{L\hat{P}I}}$$
$$=\frac{\mathrm{K\hat{P}O}}{\mathrm{L\hat{P}I}},\text{ since the angles are small,}$$
$$\therefore\ \mu=\frac{\mathrm{OK/OP}}{\mathrm{IL/IP}}.$$

But the magnification $=\frac{\mathrm{IL}}{\mathrm{OK}}$

12 → $\therefore$ magnification $=m=\frac{\mathrm{IP}\quad 1}{\mathrm{OP}}$

CONVENTION A	CONVENTION B	CONVENTION C
13 → $m=\frac{}{\mu u}$	$\therefore\ m=\frac{v}{\mu u}.$	$\therefore\ m=-\frac{v}{\mu u}.$

This result may be deduced in a similar manner for a convex surface.

Refraction through Lenses.—In the preceding cases we have considered rays of light passing from one medium to another. In the case of a lens the light passes from one medium to the other and then on into the first medium again, so that refraction at two faces must be considered. A lens is a part of a refracting medium generally enclosed

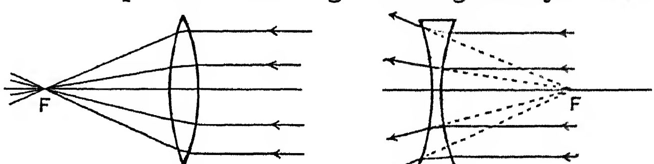


FIG. 49.

by two spherical surfaces, although in some lenses one of the faces may be plane. The optic axis of the lens is the line joining the centres of curvature of the two faces, while the focus is the point on the optic axis to which

rays from infinity converge, or from which the rays from infinity appear to diverge after passing through the lens.

Since light may pass through a lens in either direction, a lens has two foci which are equidistant from the lens and on opposite sides of it, unless the lens is a thick one.

As in the case of mirrors, lenses fall into two classes—convex or converging lenses, and concave or diverging lenses. These are so named because convex lenses cause convergence of a beam of parallel rays, while concave lenses cause parallel rays to diverge. The point to which the rays converge or from which they diverge is called the focus, and the distance of the focus from the lens is the focal length. Each class may be divided into three types,

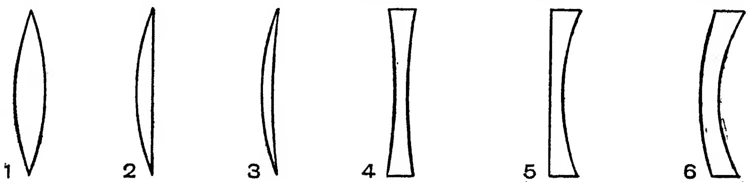


FIG. 50.

as shown: bi-convex and bi-concave, 1 and 4; plano-convex and plano-concave, 2 and 5; and convex meniscus and concave meniscus, 3 and 6.

General Equation for Lenses.—Let the lens, having a refractive index μ_2, be placed in a medium of refractive index μ_1, and let the radii of curvature of the two faces of the lens be r_1 and r_2. Also let the thickness of the lens along the axis be t. As before, we consider an object at a distance u from the lens. An image (J) of this object will be formed by refraction at the front face of the lens. Suppose this image is at a distance w from the front face of the lens. Then the ray of light reaching the other face of the lens will appear to come from J, *i.e.* from a distance $w+t$. Thus a second image (I) will be formed owing to refraction at the second face. Suppose this image is at a distance v from the second face of the lens.

Then for refraction at the first face of the lens

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\frac{\mu}{w}-\frac{1}{u}=\frac{\mu-1}{r_1}\ldots(1)$	$\frac{1}{u}=\frac{\mu-1}{r_1}\ldots(1)$	$-\frac{\mu}{w}+\frac{1}{u} \quad \mu-1 \quad (1)$ if the image is virtual,

where $\mu=\mu_2/\mu_1$.

At the second face the ray of light goes from a medium of refractive index μ_2 to one of refractive index μ_1.

We must accordingly substitute $1/\mu$ for μ in the general formula, while since the light appears to come from a distance $w+t$, u will be replaced by $w+t$. Hence at the second face we have

$$\frac{1}{v}-\frac{\mu}{w+t}=\frac{1-\mu}{r_2} \quad \Big| \quad \frac{1}{v}-\frac{\mu}{w+t}=\frac{1-\mu}{r_2} \quad \Big| \quad \frac{1}{v}+\frac{\mu}{w+t}=-\frac{1-\mu}{r_2}$$

[Note that in **C** a minus sign appears on the right-hand side of this equation; the reason is that if r_2 is positive when considered from outside the lens it is negative when considered from inside the lens (and *vice versa*) in the direction in which the light is passing.*]

For the present we will concern ourselves only with thin lenses, so that t may be neglected in comparison with w. Thus

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\frac{1}{v}-\frac{\mu}{w}=1-\mu\ldots(2)$	$\frac{\mu}{w}=1-\mu\ldots(2)$	$\frac{1}{v}+\frac{\mu}{w}=-\frac{1-\mu}{r_2}\ldots(2)$

We may now eliminate w from the two equations (1) and (2) by addition. Then

$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\ldots(3)$	$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\ldots(3)$	$\frac{1}{v}+\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\ldots(3)$

* Consider a bi-convex glass lens in air. Although both faces have positive radii of curvature when considered from the air, the radius of the second face is now taken as negative since a ray of light approaches this face through the glass of the lens.

This equation is of great importance and immediately leads us to a value for the focal length of a lens in terms of its refractive index and its radii of curvature. When the object is at infinity, $1/u$ equals zero and v equals f, the focal length. Thus

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right.$ . . . (4)	$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right.$ . . . (4)	$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right.$ . . . (4)

For a bi-convex lens in air

r_1 is negative and r_2 positive, $\therefore f$ is negative.	r_1 is positive and r_2 negative, $\therefore f$ is positive.	r_1 and r_2 are both positive, $\therefore f$ is positive.

For a bi-concave lens in air

r_1 is positive and r_2 negative, $\therefore f$ is positive.	r_1 is negative and r_2 positive, $\therefore f$ is negative.	r_1 and r_2 are both negative, $\therefore f$ is negative.

Care must be taken in referring to a lens as a positive or a negative lens since the sign of the focal length depends not only on the lens itself but also on the surrounding medium.

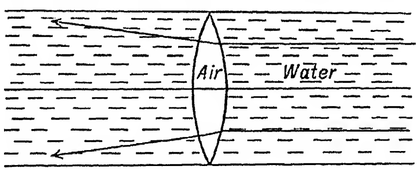


FIG. 51.

Thus in equation (4) μ is actually equal to μ_2/μ_1, where μ_1 is the refractive index of the medium surrounding the lens. If a concave lens is placed in a medium of greater refractive index, then μ becomes less than one, and the lens acts as a converging lens. Also if a hollow glass vessel in the form

of a convex lens, fig. 51, is placed in water it causes a parallel beam to diverge just as a concave glass lens acts in air.

Equations (3) and (4) may be compared and the relationship found between u, v, and f for lenses. Thus

CONVENTION A	CONVENTION B	CONVENTION C
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$	$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$	$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

—the simple equation for thin lenses.

Graphical Construction.—As in the case of mirrors, a graphical construction enables us to determine the position of the image when the object does not lie entirely on the axis. In this case we consider two rays from any point on the object, one being parallel to the optic axis and passing through or appearing to diverge from the focus after refraction through the lens, the other ray passing through the centre of the lens and being undeviated. In considering this second ray to be undeviated we are making a very slight assumption which is only approximately true. It has been shown in Chapter II that a ray passing through a medium, the sides of which are parallel, suffers a slight lateral displacement, but is unchanged in direction. In the case of a lens the central parts of each face are almost parallel, and there is thus a slight lateral displacement for all rays except the one along the optic axis. Thus in the diagram the tangents at B and C are parallel, and the path of the ray is ABCD. Since in nearly every case the ray makes only a small angle with the optic axis, the lateral displacement is so small that it may be neglected.

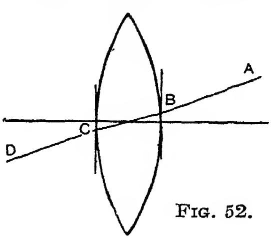


FIG. 52.

Thus in the case of the convex lens, if OK (fig. 53) is the object we have one ray KA parallel to OC, the optic axis,

passing through F after refraction, while the other ray passes undeviated along KC. The image of K is thus at L where the two rays meet.

If X is any point on OK then the image of X must lie at some point Y on IL where IL is perpendicular to the axis,

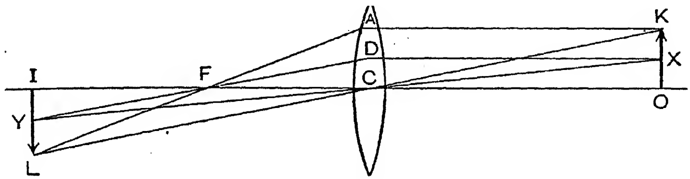


FIG. 53.

provided that OK itself is perpendicular to the axis. The rays drawn are XD parallel to the axis—the transmitted ray passing through F, and XC which is undeviated by the lens. These transmitted rays meet at Y on IL, since

$$\frac{IY}{CD}=\frac{IL}{CA}$$

for the first ray, and

$$\frac{IY}{OX}=\frac{IL}{OK}$$

for the second ray.

These expressions are identical as $CD=OX$ and $CA=OK$, thus indicating that both the transmitted rays pass through the same point Y on IL. In the same manner the image of any other point on OK is found to lie on IL, so that IL is the image of OK.

It is assumed here that all rays parallel to the axis pass through F after refraction, irrespective of their distances from the axis. This assumption is only approximate, as is shown in Chapter IV, and actually the image IL is slightly curved.

The General Equation.—This equation may be obtained easily from the diagrams constructed graphically. The two

types of lenses, convex and concave, are shown. In each case there are two sets of similar triangles, COK and CIL,

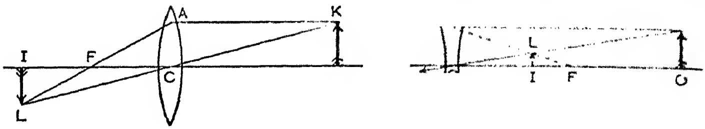


Fig. 54.

and FCA and FIL. Hence

$$\frac{OK}{IL}=\frac{CO}{CI} \quad \text{and} \quad \frac{CA}{IL}=\frac{CF}{IF}.$$

Now

$$OK=CA,$$

so that

$$\frac{CO}{CI}=\frac{CF}{IF}.$$

Substituting, with due regard to sign, we obtain

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\frac{u}{-v}=\frac{-f}{-(v-f)}$	$\frac{-u}{v}=\frac{f}{v-f}$	$\frac{u}{v}=\frac{f}{v-f}$

for the convex lens, and

$$\frac{u}{v}=\frac{f}{f-v} \qquad \frac{-u}{-v}=\frac{-f}{-(f-v)} \qquad \frac{u}{-v}=\frac{-f}{-(f-v)}$$

for the concave lens.

In both cases the equation is

$$uf-uv=vf. \quad \Big| \quad uf-uv=vf. \quad \Big| \quad uv-uf=vf.$$

Dividing by uvf, we have

$$\frac{1}{v}-\frac{1}{f}=\frac{1}{u}, \qquad \frac{1}{v}-\frac{1}{f}=\frac{1}{u}, \qquad \frac{1}{f}-\frac{1}{v}=\frac{1}{u},$$

or, in the more familiar form,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}. \qquad \frac{1}{v}-\frac{1}{u}=\frac{1}{f}. \qquad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

The Deviation Method.—The general equations for lenses may be derived more rapidly by use of the deviation method. A lens can be treated as a series of prisms of small angles, these angles increasing in size from the centre of the lens outwards. For a prism of small angle A, the deviation D is given by $D=(\mu-1)A$ (page 142), where μ is the refractive index. This equation is only true when the angle of incidence is small, but such is always the case with ordinary lenses in practice. For a ray initially

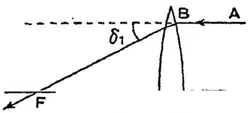


FIG. 54 (i).

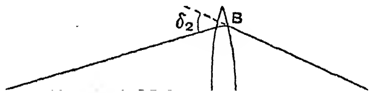


FIG. 54 (ii).

parallel to the axis the deviation δ_1 produced is h/f (neglecting signs), where h is the distance of the point of incidence from the pole of the lens. This is illustrated by fig. 54 (i), where AB is the incident ray and F the focus.

For a ray from an object O, fig. 54 (ii), incident at the same point on this lens the deviation is δ_2 and the emergent ray cuts the axis at I. Now the deviation is independent of the angle of incidence (provided this is small), so that $\delta_1=\delta_2$. The values of δ_1 and δ_2 can now be found and equated for the case illustrated.

Since $\delta_2=B\hat{O}I+B\hat{I}O$, we have

CONVENTION A	CONVENTION B	CONVENTION C
$\delta_1=-h/f,$	$\delta_1=h/f,$	$\delta_1=h/f,$
$\delta_2=h/u-h/v,$	$\delta_2=h/v-h/u,$	$\delta_2=h/v+h/u,$
$\therefore\ \frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$	$\therefore\ \frac{1}{v}-\frac{1}{u}=\frac{1}{f}.$	$\therefore\ \frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$

The other lens equation is deduced in a similar manner. Let the angle between the faces of the lens at a distance h from the axis be A. This angle is equal to the angle

between the radii drawn to the lens faces at this distance from the axis. Thus in fig. 54 (iii), $A = \theta + \phi$.

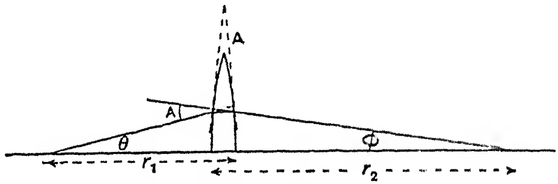


FIG. 54 (iii).

Then, taking D as the deviation, and r_1 and r_2 as the radii of curvature, we have

CONVENTION A	CONVENTION B	CONVENTION C
$D = -h/f$	$D = h/f$	$D = h/f$
$\theta = -h/r_1$	$\theta = h/r_1$	$\theta = h/r_1$
$\phi = h/r_2.$	$\phi = -h/r_2.$	$\phi = h/r_2.$

$$\text{But } D = (\mu - 1)\,A = (\mu - 1)(\theta + \phi).$$

$$\therefore -\frac{h}{f} = (\mu - 1)\left(\frac{h}{r_2} - \frac{h}{r_1}\right) \qquad \therefore \frac{h}{f} = (\mu - 1)\left(\frac{h}{r_1} - \frac{h}{r_2}\right) \qquad \therefore \frac{h}{f} = (\mu - 1)\left(\frac{h}{r_1} + \frac{h}{r_2}\right)$$

$$\text{or } \frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \qquad \text{or } \frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad \text{or } \frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Magnification.—The linear magnification, m, produced by a lens may be expressed in terms of u and v. From fig. 54 we have

$$m = \frac{IL}{OK} = \frac{CI}{CO}.$$

Remembering that m is positive for an erect image and negative for an inverted image, we may write

CONVENTIONS **A** and **B**

$$m = \frac{v}{u}.$$

This equation is opposite in sign to the corresponding one for mirrors. The reason is that u and v have the same sign for a real image formed by a mirror, but opposite signs for a real image formed by a lens.

CONVENTION **C**

$$m = -\frac{v}{u}.$$

This equation is exactly the same as the one obtained for mirrors. In each case the image is erect when virtual (v negative) and inverted when real (v positive).

A classification of the position and nature of the image for different positions of the object can now be made.

Position of object	Position of image	Nature of image
(a) *Convex Lenses*		
At infinity	At focus	Real
u greater than twice focal length	$v >$ focal length but $<$ twice focal length	Real, inverted, diminished
u equal to twice focal length	v equal to twice focal length	Real, inverted, same size
$u >$ focal length but $<$ twice focal length	v greater than twice focal length	Real, inverted, magnified
At focus	At infinity	
Between focus and pole of lens	Between infinity and lens	Virtual, erect, magnified
At pole	At pole	Erect, same size
(b) *Concave Lenses*		
At infinity	At focus	Virtual
Between infinity and pole of lens	Between focus and pole of lens	Virtual, erect, diminished
At pole	At pole	Erect, same size

The student should compare the classification for mirrors with that given above for lenses. Certain points of similarity will be noticed between concave mirrors and convex lenses. For the object more distant than the focus

the image is always real and inverted, while for the object nearer than the focus the image is virtual, erect and magnified. In each case when the object is distant by twice the focal length from the mirror or lens the magnification is unity. When a real image is formed it is always possible to interchange the object and image. In such cases the positions of object and image are known as conjugate foci.

Similarities also exist between convex mirrors and concave lenses. Here the image is always virtual, erect and diminished for all positions of the object except $u=0$, when the magnification becomes unity.

Focal Length of Two Lenses in Contact.—Let the two lenses have focal lengths f_1 and f_2, their axes being coincident. Then an image of an object, distance u away, is formed by refraction through the first lens. Let the distance of the image from the lens be v_1. Then

CONVENTION A	CONVENTION B	CONVENTION C
$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}$ . . . (1)	$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1}$ . . . (1)	* $\frac{1}{v_1}+\frac{1}{u}=\frac{1}{f_1}$ . . . (1)

To the second lens the object appears to be where this image is formed, and since the lenses are thin and in contact, the final image is formed at a distance v from the lens where

$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2}$ . . . (2)	$\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2}$ . . . (2)	$\frac{1}{v_1}=\frac{1}{f_2}$. (2)

By addition v_1 is eliminated, and

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}.$	$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}.$	$\frac{1}{v}+\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}.$
But $\frac{1}{v}-\frac{1}{u}=\frac{1}{\mathrm{F}}$	But $\frac{1}{v}-\frac{1}{u}=\frac{1}{\mathrm{F}}$	But $\frac{1}{v}+\frac{1}{u}=\frac{1}{\mathrm{F}}$

* If the image formed by the first lens is real, v_1 is positive. Such an image acts as a virtual object to the second lens so that $-v_1$ replaces u in the general equation for lenses, as in (2).

where F is the focal length of the equivalent lens.

$$\therefore \frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

The *deviation method* yields the same result.

For a ray meeting the lens at a distance h from the axis the deviation produced is h/f. When we have two lenses in contact, the total deviation is the sum of the deviations produced by each lens.

$$\frac{h}{F}=\frac{h}{f_1}+\frac{h}{f_2}$$

or

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

The reciprocal of the focal length is called the power of the lens, and consequently the resultant power of two thin lenses in contact is the algebraic sum of their respective powers.

Focal Length of a Lens System.—When two lenses having the same axis are separated, the resultant focal length depends on the distance apart of the lenses as well as on the individual focal lengths.

Let AB and CD be two concave lenses at a distance apart a, and let their foci be F_1 and F_2 respectively.

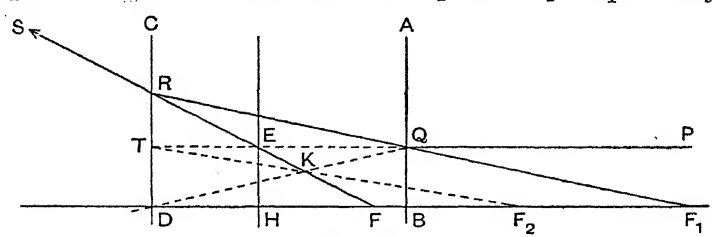


FIG. 55.

Then a ray of light PQ, parallel to the axis, is deviated by the first lens and travels in a direction QR, appearing

to have come from F_1. In order to find the direction of the ray after passing through the second lens we must obtain the position of the image of Q formed by the lens CD. By the usual construction this is found to be at K, so that the emergent ray is in the direction KRS. The actual path of the ray is thus along PQRS. The focus is F where the emergent ray, produced back, cuts the axis.

This arrangement is similar to a single concave lens placed at EH and having a focal length HF. Thus the equivalent focal length of the system is HF.

Now

$$\frac{TE}{DF}=\frac{TQ}{DF_1},$$

$\frac{RT}{RD}$ Δs RET, RFD Δs RQT, RFD

and

$$\frac{EQ}{DF}=\frac{TQ}{DF_2}.$$

By addition

$$\frac{TE+EQ}{DF}=\frac{TQ}{DF_1}+\frac{TQ}{DF_2},$$

or

$$\frac{\ }{DF}=\frac{\ }{DF_1}+\frac{\ }{DF_2},$$

$$DF=\frac{DF_1 . DF_2}{DF_1+DF_2}.$$

Now

$$HF=DF-DH=DF-TE.$$

But

$$TE=DF.\frac{TQ}{DF_1},$$

$$HF=DF\left(1-\frac{TQ}{DF_1}\right)$$

$$=\frac{DF_1 . DF_2}{DF_1+DF_2}\left(\frac{DF_1-TQ}{DF_1}\right).$$

Substituting the correct symbols, we obtain

CONVENTION A	CONVENTION B	CONVENTION C
$\mathrm{HF}=\dfrac{f_2 f_1}{(a+f_1+f_2)}$.	$\mathrm{HF}=\dfrac{f_2 f_1}{(a-f_1-f_2)}$.	$\mathrm{HF}=\dfrac{f_2 f_1}{(a-f_1-f_2)}$.
But HF = F, the equivalent focal length of the system.	But HF = − F, the equivalent focal length of the system.	But HF = − F, the equivalent focal length of the system.
$\therefore\ \mathrm{F}=\dfrac{f_1 f_2}{(f_1+f_2+a)}$.	$\therefore\ \mathrm{F}=\dfrac{f_1 f_2}{(f_1+f_2-a)}$.	$\mathrm{F}=\dfrac{f_1 f_2}{(f_1+f_2-a)}$.

This may be written in the form

$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1 f_2}, \qquad \frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1 f_2}, \qquad \frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1 f_2},$$

so that when a is zero (*i.e.* when the lenses are in contact)

$$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}.$$

The *deviation method* may be used here instead of the above. By fig. 55 we have

$$\begin{aligned}
&\text{deviation at first lens} &&=\mathrm{BQ}/\mathrm{BF}_1,\\
&\text{deviation at second lens} &&=\mathrm{DR}/\mathrm{DF}_2,\\
&\text{deviation at equivalent lens} &&=\mathrm{HE}/\mathrm{HF}.
\end{aligned}$$

Now

$$\mathrm{DR}=\mathrm{DT}+\mathrm{TR}=\mathrm{BQ}+\mathrm{TR},$$

and

$$\begin{aligned}
\mathrm{TR}&=\mathrm{TQ}\tan \mathrm{R\hat{Q}T}\\
&=\mathrm{TQ}\tan \mathrm{Q\hat{F}_1B}\\
&=\mathrm{TQ}\frac{\mathrm{BQ}}{\mathrm{BF}_1}.
\end{aligned}$$

$$\therefore\ \mathrm{DR}=\mathrm{BQ}\left(1+\frac{\mathrm{TQ}}{\mathrm{BF}_1}\right).$$

Hence, since the deviation at the equivalent lens equals the sum of the deviations at each lens of the system,

$$\frac{\mathrm{HE}}{\mathrm{HF}}=\frac{\mathrm{BQ}}{\mathrm{BF}_1}+\frac{\mathrm{DR}}{\mathrm{DF}_2}$$

$$=\frac{\mathrm{BQ}}{\mathrm{BF}_1}+\frac{\mathrm{BQ}}{\mathrm{DF}_2}\left(1+\frac{\mathrm{TQ}}{\mathrm{BF}_1}\right)$$

or

$$\frac{1}{\mathrm{HF}}=\frac{1}{\mathrm{BF}_1}+\frac{\mathrm{BF}_1+\mathrm{TQ}}{\mathrm{DF}_2.\mathrm{BF}_1}$$

$$=\frac{1}{\mathrm{BF}_1}+\frac{1}{\mathrm{DF}_2}+\frac{\mathrm{TQ}}{\mathrm{DF}_2.\mathrm{BF}_1}.$$

CONVENTION A	CONVENTION B	CONVENTION C
$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1f_2}.$	$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1f_2}.$	$\frac{1}{\mathrm{F}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{a}{f_1f_2}$

Determination of the Focal Lengths of Mirrors and Lenses.—In most experiments for finding the constants of mirrors and lenses the optical bench is used. This bench consists of a wooden or metal base, along one edge of which is fastened a scale two metres long. Stands carrying lenses or mirrors can be moved backwards or forwards along the bench, one edge of the stand always being in contact with the scale. A pointer on the stand indicates its position on the scale. The object is usually two wires at right angles, stretched across a small circular hole in a screen, and illuminated from behind by an opal or pearl electric lamp. The screen may be a sheet of white paper or a piece of ground glass. Although the position of the stand on the scale can be read off, the position of the optical centre of the lens or mirror—from which measurements have to be made—with regard to the stand may not be known. Consequently a rod of known length, 10 or 20 centimetres, is required for measuring distances. Then if the distance between a mirror and an object is required, the position of the object on the scale is noted, and it is then moved either

towards or away from the mirror until the rod of known length exactly fits in between them, one end being just in contact with the mirror and the other with the object. The position of the object on the scale is again noted, and the distance it has been moved is either added to or subtracted from the length of the rod. In this manner the distances between the different pieces of apparatus may be found accurately.

It must be noted that good results cannot be obtained unless care is taken to secure alignment between the pieces of apparatus used on the bench. The object, whether it be the cross-wires or a pin, must be adjusted so that it is perpendicular to the optic axis of the lens or mirror, its centre being on this axis. The lens or mirror must be mounted so that its optic axis is parallel to the scale on the bench. By taking such precautions much time which is often spent in searching for images will be saved.

Also the student should remember that in the parallax methods for coincidence the coincidence is not obtained by chance but by scientific method. As the eye is moved from side to side, it is always the more distant object or image which appears to move with the eye, and the position of that object or image should be adjusted accordingly.

If experiments on mirrors are carried out, best results will be obtained when using black glass mirrors, or mirrors silvered on the front surface. In this way refraction effects in the glass of the mirror are eliminated. If black glass mirrors are used, a suitable object is a vertical iron wire heated to redness in a bunsen flame.

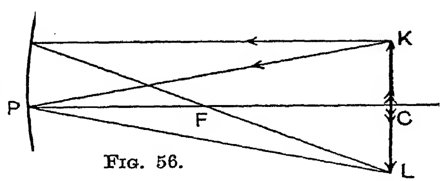


Fig. 56.

Focal Length of a Concave Mirror. — (1) The image is made to coincide with the object. The mirror is tilted very slightly so that the reflected rays fall on the

screen containing the cross-wires. When a sharp image is obtained the distance of the object from the mirror is measured. This distance is equal to the radius of curvature of the mirror, and so equals twice the focal length:

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r}=\frac{1}{f}.$$

But
$$v=u,$$
$$\therefore\ r=u=2f,$$

giving a value for f.

(2) The positions of the image for different positions of the object are found, and f determined from the formula $1/u+1/v=1/f$. This method may be subdivided into two classes—one in which a real image is formed, and the other in which the image is virtual. Here it is more convenient to use a large pin or knitting-needle as the object. If

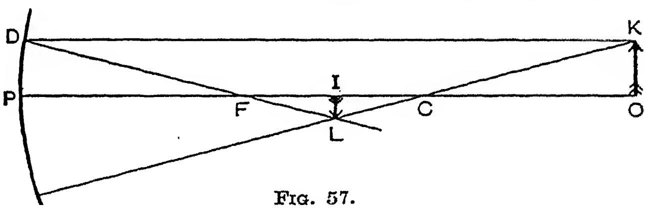


FIG. 57.

the image formed is a real one it will be in front of the mirror. So a stand containing a pin is moved backwards or forwards along the scale until the pin coincides with the image IL by reflection of the first pin or object OK. The distances u and v are then measured and f calculated.

When the image is virtual it is behind the mirror, so that the stand containing the second pin must be placed at the back of the mirror. The stand is then moved until the part of the pin seen above the mirror appears to be a continuation of the image seen in the mirror.

In this case (virtual image) it is not always easy to measure v accurately. Better results are usually obtained

by covering the lower half of the mirror with a semi-circular plane mirror N, the image IL formed by the concave mirror being made to coincide with the image IS, of QR, formed by the plane mirror.

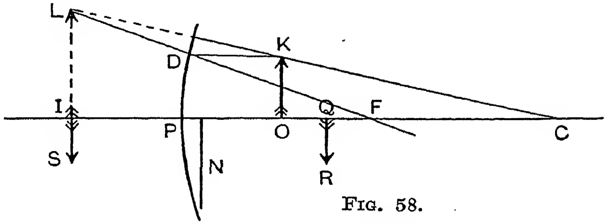


FIG. 58.

The image IS is as far behind N as QR is in front, so that the position of IL may be found.

Focal Length of a Convex Mirror.—(1) The image formed by a convex mirror is always virtual. Consequently we may employ a method entirely similar to the one used for concave mirrors when the image is virtual.

The distances of the object and image from the mirror are measured, and the focal length determined from the formula $1/u + 1/v = 1/f$.

(2) An object is made to appear virtual, *i.e.* behind the mirror, so that the image is real. A convex lens L is

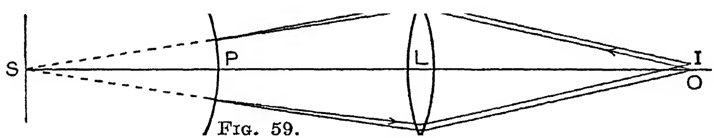


FIG. 59.

needed in this experiment, and it is placed so that a real image of the cross-wires is formed on the screen at S. Without moving any of the stands the convex mirror is then placed between L and S so that it faces the cross-wires. Rays falling on the mirror are reflected back, and if they meet the mirror normally they go back along the same path and form an image at the cross-wires. The

mirror is tilted very slightly so that the image falls on the screen containing the wires at I, and not on the wires themselves. The stand containing the mirror is moved until the image is sharp, and the distance of the screen S from the mirror is measured. It is obvious that this distance is equal to the radius of curvature of the mirror since the rays strike the mirror normally.

Focal Length of a Convex Lens.—(1) A plane mirror is fixed in a stand behind the lens so that rays passing through the lens are reflected back again. The stand containing the lens is moved backwards or forwards along the bench until a sharp image of the cross-wires is formed on the screen containing the wires. When this happens

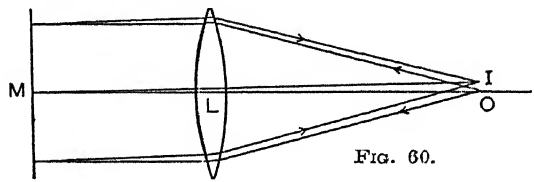


FIG. 60.

the rays passing through the lens must meet the mirror at right angles and travel back along the same path. Since these rays meet the mirror at right angles they must be parallel. But parallel rays on passing through a convex lens are all converged to the focus, and so in the experiment the cross-wires must be at the focus of the lens. The distance between the lens and cross-wires is thus measured, and gives the focal length immediately.

It should be noted that the distance of the plane mirror from the lens does not affect the experiment.

(2) The real image method may be used as in the case of the concave mirror. A screen is moved so that a sharp image of the object is formed on it by the lens. The screen and the object, a lamp, are on opposite sides of the lens. The distances from the lens are noted, giving v and u respectively. Thus f is determined by the lens equation.

If the object is between the lens and its focus, the image

formed is a virtual one, so that it is advisable to use the pin method and to cover the lower half of the lens with the plane mirror N. This mirror is on the opposite side

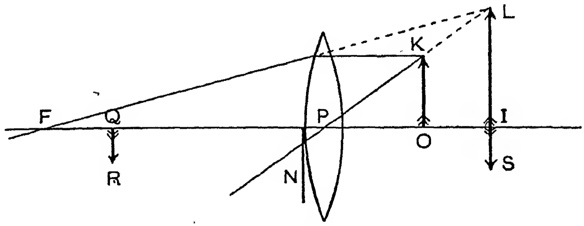


FIG. 61.

of the lens to the object. The image of the object viewed through the lens is made to coincide with the image in the plane mirror of another object QR. The position of the image IL is thus as far behind the plane mirror as QR is in front of it. Distances are measured and f obtained from the formula.

(3) The minimum distance between object and image for a real image to be formed is $4f$ numerically. This may be proved as follows:—

Let x be the distance between object and image. Then

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$x=u-v.$	$x=v-u.$	$x=u+v.$

Differentiating with regard to v,

$\frac{dx}{dv}=\frac{du}{dv}-1 \quad . (1)$	$\frac{dx}{dv}=1-\frac{du}{dv} \quad (1)$	$\frac{dx}{dv}=\frac{du}{dv}+1 \quad . . . (1)$
Also $\quad -\frac{1}{u} = \frac{1}{f},$	Also $\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$	Also $\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$
whence $\frac{dv}{v^2}-\frac{du}{u^2}=0,$	whence $\frac{dv}{v^2}-\frac{du}{u^2}=0,$	whence $\frac{dv}{v^2}+\frac{du}{u^2}=0,$
or $\quad \frac{du}{dv}=\frac{u^2}{v^2}.$	or $\quad \frac{du}{dv}=\frac{u^2}{v^2}.$	or $\quad \frac{du}{dv}=-\frac{u^2}{v^2}.$

Substitution in equation (1) for $\frac{du}{dv}$ gives

CONVENTION A	CONVENTION B	CONVENTION C
$\frac{dx}{dv}=\frac{u^2}{v^2}-1.$	$\frac{dx}{dv}=1-\frac{u^2}{v^2}.$	$\frac{dx}{dv}=1-\frac{u^2}{v^2}.$

Now for x to have maximum or minimum values,

$$\frac{dx}{dv}=0.$$

Hence

$$\frac{u^2}{v^2}=1,$$

giving

$$u=\pm v.$$

The corresponding values for x are thus 0 and $4f$. Both of these are minimum values. The value $x=0$ occurs when the object is at the lens, the image then being also at the lens. The minimum value of x for a real image is thus $4f$ numerically, and results when u and v each equal $2f$ numerically.

The experiment can be performed very simply. If the distance between the object and the screen on which the image is formed is initially greater than $4f$, there are two positions of the lens for which a sharp image is formed. The screen is moved slowly towards the lens, the position of the lens being adjusted so that the image remains sharp. When it becomes impossible to keep a sharp image the distance between object and image will be less than $4f$, and consequently a slight adjustment is necessary to make the image clear. If the position of the lens is altered it will be noticed that for no other position between object and screen is a sharp image formed. The distance between object and image is then measured and divided by four, giving a value for the focal length of the lens.

(4) Provided that the refractive index of the material of the lens is known, the focal length of the lens may be

determined by Boys's method. This entails the use of the equation

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
$\frac{1}{f}=(\mu-1)\Big(\frac{1}{\ }-\frac{1}{\ }$	$\frac{1}{f}=(\mu-1)\Big(\frac{1}{r_1}-\frac{1}{\ }$	$\frac{1}{f}=(\mu-1)\Big(\frac{1}{r_1}+\frac{1}{r_2}$

in which r_1 and r_2 have to be found. The lens L is set up vertically in front of a dark screen. A vertical iron wire P, heated in a bunsen flame, is then moved backwards or forwards along the optical bench until it coincides with

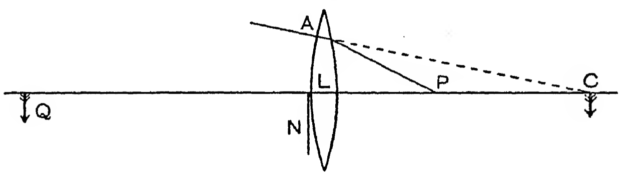


FIG. 62.

the image of itself, reflection taking place at the more distant surface of the lens. Since the ray of light is reflected back along the same path, it strikes the distant face of the lens normally. Some of the light is transmitted, and if the screen is now removed and the object observed through the lens, an image is seen. This image is at C, where CA is the normal at A to the distant face. In order to find the position of C a plane mirror N is placed to cover the lower half of the lens, and an object (a pin) Q is moved along the bench until the image of Q formed by reflection at the mirror coincides with the image of P seen through the lens. Then the distance NQ equals NC so that the position of C may be found. The distance of C from the further face of the lens is measured, giving the radius of curvature of that face of the lens.

The lens is reversed and the radius of curvature of the other face found in a similar manner. Thus r_1 and r_2 are known, and f may be determined.

In many cases this method is used for the determination

of the refractive index of the material of the lens, f being found by one of the methods given above.

✓ This method may also be used for finding the refractive index of a liquid. A little of the liquid is placed on a plane mirror M, and a convex lens L then placed on the mirror. The focal length of the combination of lenses (the glass convex lens and the liquid plano-concave lens) is found by the coincidence method, the focal length of the convex lens alone also being obtained by this method. Then, since

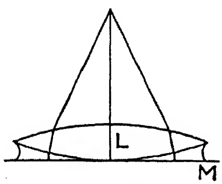


FIG. 63.

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2},$$

where F is the focal length of the combination, f_1 that of the glass lens, and f_2 that of the liquid lens, it is possible to obtain a value for f_2. But

$$\frac{1}{f_2} = \mu - 1$$

for the liquid lens, where r is the radius of curvature of its upper face. But this radius is the same as that of the lower face of the glass lens, which may be found by the method already outlined. Hence μ for the liquid can be found.

Focal Length of a Concave Lens.—(1) The image formed by a concave lens is always virtual. The focal length may

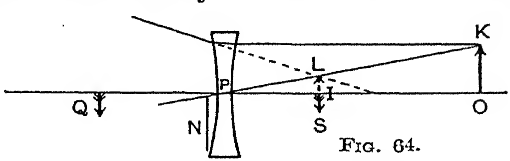


FIG. 64.

thus be found by a method similar to that for the convex lens when the image is virtual. A pin OK is observed through the lens, and a second pin Q, on the same side of

the lens as the eye, is moved until its image in the plane mirror N coincides with the image of the first pin formed by refraction through the lens.

Distances from the lens are measured and by substitution in the lens equation a value for f is obtained.

(2) As in a method for finding the focal length of a convex mirror, the object is made to appear virtual so that a real image is formed. A convex lens is placed so that a real image of the cross-wires is formed on the

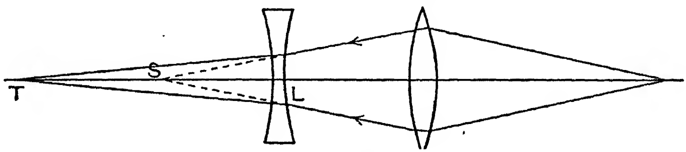


FIG. 65.

screen at S. The concave lens L is then placed between the convex lens and the screen, which now has to be moved to T in order to give a sharp image. Then the distances LS and LT are measured, giving values for u and v respectively. Care must be taken to secure the correct signs for these quantities when substitution into the lens equation is made. Thus

CONVENTION A	CONVENTION B	CONVENTION C
LS and LT are both negative.	LS and LT are both positive.	LS is negative, LT is positive.

The convex lens used must be sufficiently strong compared with the concave lens to form a real image at T. If no image is formed then the light after passing through L will be divergent, indicating that either a stronger convex lens is required or an adjustment of the distance of the object from the convex lens is needed.

(3) A convex lens and the concave lens are mounted so that they are in contact, and act as a convex lens. The focal length of the combination may be found by any of

the methods given for convex lenses. If F is this focal length, and f_1, f_2 are the focal lengths of the concave and convex lenses separately, then

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

The focal length of the auxiliary convex lens must also be found, so that a value for f_1 may be obtained.

(4) If the refractive index of the glass of the concave lens is known and the radii of curvature be found directly, then f is found from

CONVENTION A	CONVENTION B	CONVENTION C
$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$	$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$	$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

The faces of the lens act as concave mirrors, so that the radii of curvature may be found by the coincidence (of object and image) method.

The radii of curvature of spherical surfaces may be found by use of the spherometer, and this method may be used for finding the focal lengths of mirrors and lenses.

Additional methods for the determination of the focal lengths of lenses are given in Chapter V, where the theory of thick lenses and systems of lenses is given. All the foregoing experiments refer to thin lenses alone, and cannot be used for combinations of lenses.

EXAMPLES ON CHAPTER III

1. PBCA is the axis of a concave spherical mirror, A being a point object, B its image, C the centre of curvature of the mirror and P the pole. Find a relation between PA, PB and PC, supposing the aperture of the mirror to be small.

A concave mirror forms, on a screen, a real image of twice the linear dimensions of the object. Object and screen are then moved until the image is three times the size of the object. If the shift of the screen is 25 cm., determine the shift of the object and the focal length of the mirror. (N.)

2. Show that an object placed at a distance d from a concave spherical mirror of radius R gives rise to an image at a distance K, where

$$K=\frac{Rd}{2d-R}.$$

Obtain the expression from first principles, and state carefully what assumptions you make.

Describe an optical method of determining the radius of curvature of a convex spherical mirror. (N.)

3. Deduce an expression connecting the distance of an object from a concave mirror with that of its image.

It is desired to look at an image of one's own eye 10 in. away, magnified four times. Show that this can be done by means of a concave mirror, of radius of curvature $5\frac{1}{3}$ in., placed at a distance of 2 in. from the eye. (O. & C.)

4. Describe how to find the focal length of a convex spherical mirror, using a luminous object, a screen and any additional apparatus that may be necessary.

A luminous object is placed 30 cm. from the surface of a convex mirror, and a plane mirror is set so that the images formed in the two mirrors lie adjacent to each other in the same plane. If the plane mirror is then 22 cm. from the object, what is the radius of curvature of the convex mirror? (N.)

5. Describe and explain the changes which take place in the position and size of the image formed by a convex mirror as the object approaches the mirror from a great distance.

A convex mirror is usually fitted on the handlebar of a motor-cycle to give the rider a view of the road behind him. What advantage has a convex mirror over a plane mirror for this purpose? (O. & C.)

6. Describe how you would determine the focal length of a concave lens by a method involving the use of an auxiliary convex lens which is not placed in contact with the concave lens.

A thin concave lens has a focal length of 30 cm. Determine the position of the image which it forms of an object 10 cm. away (*a*) if the object is real, (*b*) if it is virtual. In each case show the path of a pencil of rays by which an eye, suitably placed, may see a non-axial point on the image. (N.)

7. Where must an object be placed with respect to a convex lens of one-inch focal length in order that (1) a real image, (2) a virtual image, may be formed a foot away from the lens? Give a diagram showing, in each case, how the image is formed. (Lond. Inter.)

8. What do you understand by an "image"? How can you decide whether it is real or virtual? An object is placed at a distance of 9 cm. from a convex lens of 6 cm. focal length. At a distance of 58 cm. from the lens on the other side is placed a concave mirror of 16 cm. radius. Find the position and nature of the final image, showing the paths of the rays on a diagram. (Camb. Schol.)

9. A metal plate containing an illuminated circular hole is placed at one end of an optical bench, and a screen at the other. By means of a convex lens an image of the hole is formed on the screen, the diameter of the image being 2·25 cm. If the lens is moved 20 cm. along the bench an image of the hole again appears on the screen, its diameter now being 1·00 cm. What is the real size of the hole and how far is it from the screen? (O. & C.)

10. What do you understand by the term "magnification" as used in optics?

The real image formed by a convex lens is twice as high as the object when the latter is 12 cm. from the lens. Where must it be placed to give an inverted image magnified four times? (O. & C.)

11. Describe a method of finding the focal length of a converging lens.

It is required to produce an image of a fixed object on a fixed screen 5 ft. away from the object, the image being four times as large as the object. What kind of lens must be used, where must it be placed and what must be its focal length? (O. & C.)

12. Establish the equation which gives the magnification produced by a thin convex lens in terms of v and u; draw careful figures illustrating the cases when the image is (a) real, (b) virtual.

A thin convex lens forms a sharp image on a screen distant l cm. from the object. The linear magnification of the image is numerically m. If m and l are varied, show that the focal length of the lens may be deduced from a graph obtained by plotting ml as ordinate against $(1+m)^2$ as abscissa. (Lond. H.S.C.)

13. Find by graphical construction the position and the size of the images of an object 1 cm. long placed 10 cm. in front of a lens of focal length 5 cm.—(a) if the lens is convex, (b) if the lens is concave. (Lond. Inter.)

14. Describe two methods for the determination of the focal length of a concave lens.

A thin equiconvex lens is placed on a horizontal plane mirror and a pin held 20 cm. vertically above the lens coincides in position with its own image. The space between the under surface of the lens and

the mirror is to be filled with water (refractive index 1·33) and then, to coincide with its image as before, the pin has to be raised until its distance from the lens is 27·5 cm. Find the radius of curvature of the surfaces of the lens. (N.)

15. Obtain a formula showing how the focal length of a thin convex lens can be calculated when the radii of its surfaces and the refractive index of the material are known. Describe how you would test the formula experimentally. (Lond. Inter.)

16. Show that the focal length of a thin lens can be calculated from a knowledge of the refractive index and the radii of curvature of the two faces.

What will be the focal length in water of a bi-convex lens whose radii of curvature are 10 cm. and 15 cm.?

(Refractive index of water 1·33, of glass 1·5.) (O. & C.)

17. A piece of wire bent into an L shape with upright and horizontal portions equal in length is placed with the horizontal portion in the axis of a concave mirror whose radius of curvature is 10 cm. If the bend is 20 cm. from the mirror, find the position of its image, and the ratio of the lengths of the images of the upright and horizontal portions of the wire. (Lond. Inter.)

18. A small linear object is placed along the axis of a convex lens of focal length 20 cm. at a distance of 50 cm. from the lens. Find the position of the image, and calculate the linear magnification produced.

19. A beam of parallel light is incident on a thin double-convex lens in a direction parallel to its axis. Some of the light is reflected from the back surface of the lens, and emerges from the front face, forming a real image. Find the distance of this image from the lens, given that each surface of the lens has a radius of curvature of 30 cm., and that the refractive index of the glass is 1·5.

(Lond. Inter.)

20. Two lenses, one convex and the other concave, each of focal length 10 cm., are placed coaxially 10 cm. apart. Calculate the position of the image of a small object placed on the common axis (*a*) 20 cm. beyond the convex lens, (*b*) 20 cm. beyond the concave lens, and give in each case a diagram showing the path of a pencil of rays from the object through the two lenses. (Lond. Inter.)

21. Show that when incident light parallel to the axis falls on the plane face of a plano-convex lens and suffers two internal reflections in the lens, the light after refraction out of the lens is focussed at a point distant F from the lens where $F = r/(3\mu - 1)$, r being the radius of the curved surface of the lens.

22. A thin convex lens of focal length 10 cm., and a convex mirror of radius 20 cm., are placed 20 cm. apart with their axes coincident. An object is placed 30 cm. from the lens on the side remote from the mirror. Calculate the position of the image and the magnification produced by the combination.

23. The plane side of a plano-convex lens is silvered, and the lens then acts as a concave mirror of 30 cm. focal length. The refractive index of the lens is 1·5. Find the radius of curvature of the convex surface.

24. Prove the formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ for lenses, the symbols having their usual significance.

A real image of a small source is produced by a convex lens at 20 cm. from the lens. A concave lens, focal length 15 cm., is introduced between the convex lens and this image, and a final real image is formed 30 cm. from the original image. Determine the position of the concave lens.

Discuss and illustrate what happens when the concave lens is moved towards the convex lens. (Lond. H.S.C.)

25. A thin plano-convex lens rests with its curved surface in contact with a horizontal plane mirror, and the space between them is filled, firstly with water, secondly with oil. A thin rod is held horizontally above the lens, and it is found that the image of the rod formed by lens and mirror coincides with the object when the distances of the rod from the lens are 20 cm. and 25 cm. respectively. If the refractive indices of water and oil are 1·33 and 1·45 respectively, calculate the focal length of the lens and the radius of curvature of its lower face. (Lond. H.S.C.)

26. A beam of light is rendered convergent by a convex lens and is brought to a focus 30 cm. from the lens. A second convex lens is now placed 10 cm. from the first one on the opposite side to the source of light, and the light is brought to a focus 4 cm. from this second lens. Find the focal length of the second lens.

27. Two convex lenses, each of focal length f, are placed at a distance $3f$ apart. For what positions of the object will a real image be formed by this combination of lenses? (Lond. Inter.)

28. A convex lens of 10 cm. focal length is held in a horizontal position just above the surface of a liquid filling a tank 20 cm. deep. The image of a point 30 cm. above the centre of the lens is brought to a focus on the bottom of the tank. Draw a diagram showing the path of the rays, and calculate the index of refraction of the liquid. (Lond. Inter.)

29. An object is placed in front of the plane face of a plano-convex lens, and coincides with its image formed by reflection at one face of the lens when 10 cm. away. A plane mirror is now placed behind the lens, and the object coincides with the image formed by light reflected from the mirror when 30 cm. from the lens. Find the radius of curvature of the curved face of the lens, and also the refractive index of the glass of the lens.

30. Obtain formulæ to give the position and magnification of the image of an object at distance u from a thin double-convex lens of refractive index μ, the radii of the two surfaces being r and s.

Two such lenses are placed at a distance a apart. Find the positions of the principal foci of the combination. (Camb. Schol.)

31. Show that, in general, there are two coaxial positions of a convergent lens which will give, on a fixed screen, a sharp image of a fixed object.

If the distance between object and screen is 96 cm., and the ratio of the lengths of the two images 4·84, what is the focal length of the lens? (N.)

CHAPTER IV

SPHERICAL AND CHROMATIC ABERRATION

Spherical Aberration.—In deriving the general formula for mirrors it was assumed that the angles between the incident rays or the reflected rays and the optic axis were small. This assumption is not justified in all cases, especially when the curvature of the mirror is large and the object is near to the mirror. If MN represents a concave mirror, O being the object, then it is found that rays reflected at the peripheral portions of the mirror cut the optic axis nearer to the pole of the mirror than do any

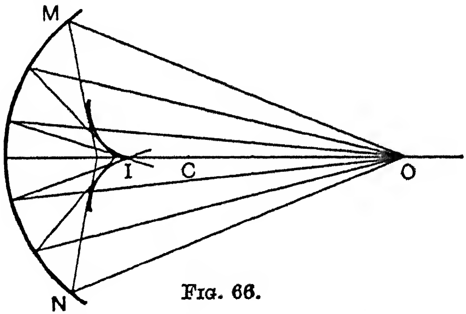


Fig. 66.

other reflected rays. The rays reflected from the central portions of the mirror form an image of O at I. It is noticed that a curve can be drawn touching each reflected ray. This is known as a *caustic curve*, and has a cusp at I. The type of curve is probably familiar, since it may be seen easily on the surface of tea in a cup, the light being reflected from the inside of the cup. This effect is known as spherical aberration. Generally, the spherical aberration of a mirror is taken as the distance between the focus for rays near to the axis and the focus for rays which are reflected at the edge of the mirror. In the diagram the

rays reflected from the mirror are only drawn in one plane —the plane of the paper. Now the mirror is part of a sphere, and so the caustic curve is only part of a surface which the reflected rays from all parts of the mirror touch. This surface is called the caustic surface, and is the surface traced out by the caustic curve if we imagine the diagram to be rotated about the optic axis.

If a diverging pencil of rays falls on a mirror, then after reflection the pencil converges to two lines. Let MP represent part of the mirror, and consider two rays OA and OB diverging from O and meeting the mirror at A and B. After reflection let these rays cut each other at D, and meet the axis at E and F respectively. Then if we consider

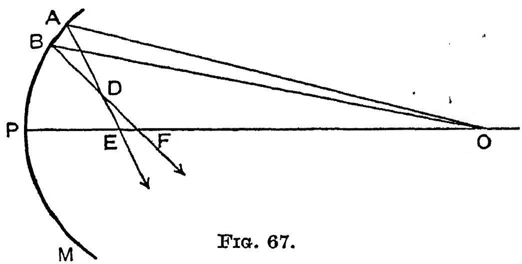


Fig. 67.

the diagram to be rotated through a small angle about the optic axis the triangle OAB traces out the pencil of rays diverging from O. *Also the point D traces out a short* line DD′ through which all the reflected rays pass. The reflected rays also pass through the line EF, so that the reflected pencil converges to a line DD′ perpendicular to the plane of the paper and to a line EF along the optic axis. These are called focal lines, DD′ being the first focal line and EF the second focal line. At neither of these positions is a clear image formed, but at a position between these lines the image becomes most distinct. Here the width and depth of the beam become equal, and since the image is approximately circular, it is called the *circle of least confusion.*

We may consider in another manner the formation of the circle of least confusion. Let AB and CD be rays reflected at the outer parts of a mirror, and let these cut the axis at E. Then all other reflected rays cut the axis at points between E and I, where I is the position of the image due to the rays reflected from the central part of the mirror, or is the position of the cusp of the caustic curve. If a screen is moved along the axis then it is obvious

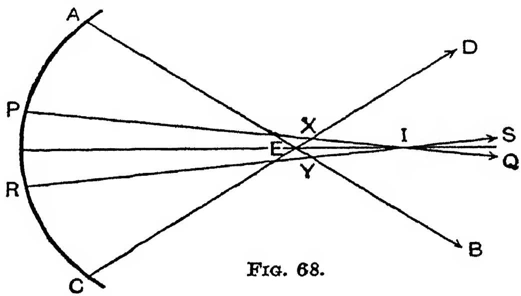


Fig. 68.

that the image will be smallest at the points of intersection of the rays AB and CD with the rays PQ and RS at XY. Nearer to the mirror the image would be a disc with a bright edge, while farther away the image would be a disc with a bright central spot. The circle formed on the screen at XY is the circle of least confusion, while EI is a measure of the spherical aberration.

The treatment in the case of lenses is exactly similar.

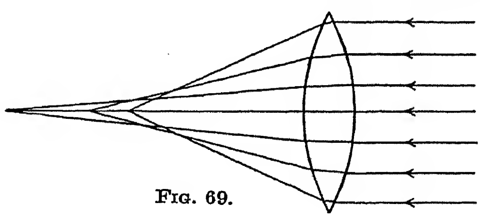


Fig. 69.

It was assumed that the angles of incidence and refraction were so small that the sines of the angles could be replaced

by the angles. For larger angles this causes an error, and rays refracted at the peripheral portions of lenses form an image nearer to the lens than do any other rays. If the outer parts of a lens are blocked out by a stop, then the lens has a longer focal length than if the central parts are blocked out. As in the case for mirrors, all the rays touch a curve after passing through the lens, the curve being called a caustic. Also focal lines and the circle of least confusion are formed in exactly the same manner.

Curvature and Distortion.—It has been shown that an object perpendicular to the axis gives rise to an image which is also perpendicular to the axis. This, however, is only an approximation—the curvature of the mirror or lens being neglected. If a careful diagram is made, it is found that the image is curved and also distorted. Taking the case of a convex lens used as a simple magnifying glass familiar example of curvature and distortion—we find

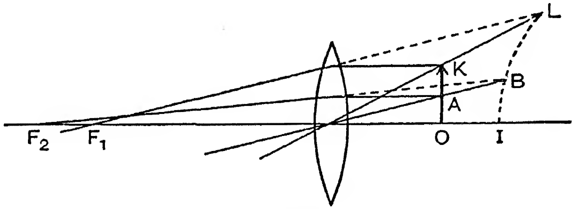


FIG. 70.

that while the image of K lies at L, the image of A, the midpoint of OK, is at B. These images are found by the usual graphical construction, and the image of OAK is thus IBL, which is seen to be curved. Moreover, BL is greater than IB, since the rays which pass through the outer portions of the lens have a different focus from those which pass through the central parts. These foci are shown at F_1 and F_2 respectively in the diagram.

This effect when the image is virtual may be seen very easily if a series of parallel lines is viewed through a convex lens of short focal length. The lines appear to curve away

from each other near the edges of the lens, and are only parallel over quite a small central part.

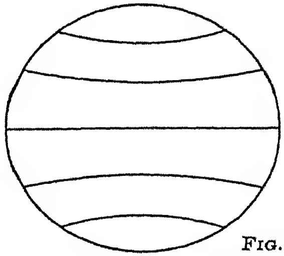


FIG. 71.

The opposite effect—where the central parts are magnified more than the outer ones—occurs when a real image is formed. In the diagram, IBL is the image of OAK. If several parallel lines are drawn on a ground glass screen which is illuminated, then the image of this formed by a

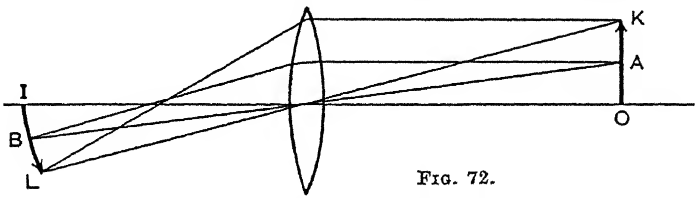


FIG. 72.

convex lens of short focal length can be shown on a

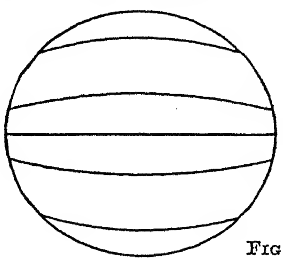


FIG. 73.

screen, and consists of lines which curve towards each other at the edges.

Similar effects may be observed in the case of spherical mirrors of large curvature.

Methods of reducing Spherical Aberration.—It is not possible to construct a mirror which will bring rays from any point on the axis to a point image. But a mirror can be constructed so that rays from some definite point on the axis form a point image after reflection. No matter where the rays meet the mirror, the total length of path is always the same. The reflecting surface is then called aplanatic. A surface having this property is an ellipsoid, and if ABC represents an ellipsoidal mirror, of which F_1 and F_2 are the foci, then all rays diverging from F_1 pass through F_2 after reflection, the length of path being constant.

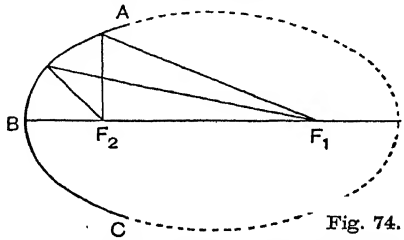


Fig. 74.

In astronomical telescopes concave mirrors are sometimes employed, but these are made in a paraboloid form instead of being spherical. For a parabola is merely an approximation to an ellipse with a very large distance between the foci. Thus a paraboloidal mirror will bring all rays from an object at infinity to a point focus.

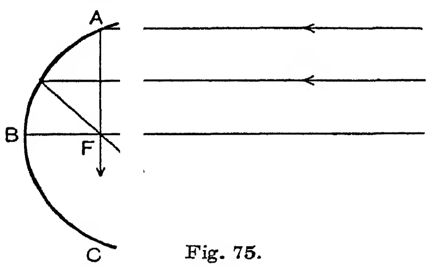


Fig. 75.

In the photographic camera the area in use of the lens can be controlled by stops. The sharpest image is formed when the central parts of the lens only are used. In order to admit more light the stop opening may be enlarged so that the exposure will be shorter, but this means that the definition of the image will not be as good. The diameter

of the stop opening is expressed in terms of the focal length of the lens, so that $f/11$ means that the diameter is 1/11 of the focal length. Thus by use of stops spherical aberration may be much reduced, although at the same time the image loses in brightness. Spherical aberration may also be diminished by using two lenses instead of one. Then the deviation which would occur at two refractions now occurs at four refractions. Consequently the angles of incidence and refraction are smaller in the second case, making the approximations mentioned before more nearly correct. Thus in the lantern condenser we have two plano-convex lenses, the curved faces being towards each other. Then deviation occurs at each of the four faces instead of at two as in the case of a single lens. The parallel rays should always be made to fall on the curved face and not the plane one, since otherwise all the deviation would occur at the curved face only.

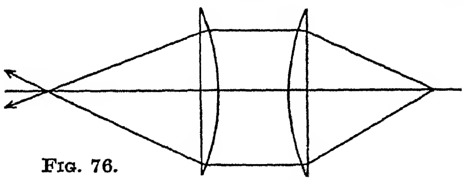


Fig. 76.

It may be wondered why lenses for telescopes should be made of large diameters. The reason for this is discussed in the chapter on diffraction, where it is shown that it is better to block out the central portions of the lens rather than the outer portions. The large diameter enables the telescope to "separate" objects which are extremely close together.

Astigmatic Lenses.—Spherical aberration may be illustrated very simply by an astigmatic lens. Such a lens has two focal lengths, one of which may, in certain cases, be infinity. Generally, one surface of the lens is spherical while the other is cylindrical, but if both faces are cylindrical and the axes of the cylinders are parallel, then it can be seen that rays in the plane containing these axes will be undeviated. Suppose we consider a lens having one

side convex and spherical, while the other side is convex and cylindrical, then in the plane containing the axis of the cylinder, the lens acts as plano-convex, while in the plane through the lens at right angles to this axis the lens acts as bi-convex. If the object is an illuminated circular aperture with cross-wires—one parallel to the axis of the cylindrical face and the other at right angles—there will be two positions of the screen for which a clear image is formed. At each position one of the cross-wires only will be seen, depending on whether the lens is acting as bi-convex or plano-convex. Midway between these images will be another image, this one being smaller than either of the others. This is the circle of least confusion, and at this position the rays of light are nearest together.

Chromatic Aberration.—If white light falls on a convex lens, and the image is observed carefully, it is noticed that if the screen is nearer to the lens than is the position of the image, the edge of the image has a reddish tint, and if the screen is more distant, the edge is tinted blue. Again, if red and blue lights are used in turn, and the focal length

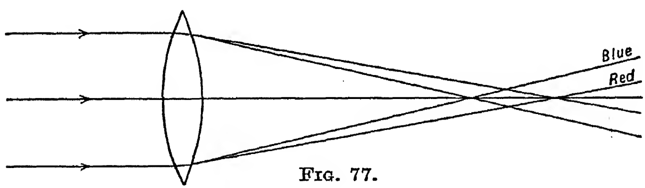


FIG. 77.

of the lens found in each case, results show that the focal length is greater for red than for blue light. This effect is known as chromatic aberration, and is due to the fact that the refractive index of a substance varies according to the colour or the wave-length of the incident light. The refractive index increases as the wave-length decreases, so that it increases as we pass from red to blue. In practice the eye automatically takes the position of the brightest image, which is intermediate between the red and the blue,

generally where the image is tinted slightly yellow. For all positions, however, the image is blurred, since the focal length is different for each different colour. Thus chromatic aberration results.

The chromatic aberration of a lens may be defined as the distance between the foci for red and blue lights.

The relationship between the focal length and refractive index is

USING CONVENTION A

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

USING CONVENTION B

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right).$$

USING CONVENTION C

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right).$$

The variables are f and μ so that differentiation gives

Convention A:

$$-\frac{df}{f^2}=d\mu\left(\frac{1}{r_1}-\frac{1}{r_2}\right),$$

or

$$\frac{df}{f^2}=\frac{d\mu}{f(\mu-1)}$$

since

$$\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{f(\mu-1)}.$$

$$-\frac{df}{f}=\frac{d\mu}{\mu-1}.$$

Convention B:

$$-\frac{df}{f^2}=d\mu\left(\frac{1}{r_1}-\frac{1}{r_2}\right),$$

or

$$\frac{df}{f^2}=\frac{d\mu}{f(\mu-1)}$$

since

$$\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{f(\mu-1)}.$$

$$-\frac{df}{f}=\frac{d\mu}{\mu-1}.$$

Convention C:

$$-\frac{df}{f^2}=d\mu\left(\frac{1}{r_1}+\frac{1}{r_2}\right),$$

or

$$\frac{df}{f^2}=\frac{d\mu}{f(\mu-1)}$$

since

$$\left(\frac{1}{r_1}+\frac{1}{r_2}\right)=\frac{1}{f(\mu-1)}.$$

$$\frac{df}{f}=\frac{d\mu}{\mu-1}.$$

The term $\dfrac{d\mu}{(\mu-1)}$ is called the *dispersive power* of the material of which the lens is made. Since this is equal to $-\dfrac{df}{f}$, a value may be found simply by experiment. The negative sign denotes that as the value of μ increases then the value of f decreases. If a source of white light is covered with first a red filter and then a blue one the focal length of the lens can be found in each case, and the dispersive power deduced by dividing the difference in the focal lengths by the mean focal length.

The more usual method of determining the dispersive power of a substance is to find its refractive indices for two, or more, wave-lengths.

Often the C and F hydrogen lines are used, so that the dispersive power may be written as

$$\frac{\mu_F - \mu_C}{\mu - 1},$$

where

$$\mu = \mu_F + \mu_C$$

Sometimes the refractive index for sodium light, the D line, is taken for the mean refractive index, in which case the dispersive power is written as

$$\frac{\mu_F - \mu_C}{\mu_D - 1}.$$

So dispersive power may be defined as the difference between the refractive indices for any two wave-lengths, divided by the mean refractive index minus one.

E.g.

Crown glass $\mu_F = 1{\cdot}521$, $\mu_C = 1{\cdot}513$,
Double dense flint glass $\mu_F = 1{\cdot}721$, $\mu_C = 1{\cdot}697$.

Dispersive power for crown glass

$$= \frac{\cdot 008}{\cdot 517}$$

$$= \cdot 0155.$$

Dispersive power for flint glass

$$= \frac{\cdot 024}{\cdot 709}$$

$$= \cdot 0339.$$

Thus the dispersive power of flint glass is about twice that for crown glass.

Achromatic Combinations of Lenses.—By suitably combining two lenses of different types of glass it is possible to produce a resultant lens which is free from chromatic aberration over certain wave-lengths. Such a combination is said to be achromatic for the range of wave-lengths.

(*a*) Lenses in contact.

Let the focal lengths of the constituent lenses be f_1 and f_2, and let the dispersive powers be denoted by D_1 and D_2 respectively. Then if the focal length of the combination is F, we have

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2},$$

which, differentiated, gives

$$\frac{dF}{F^2}=\frac{df_1}{f_1^2}+\frac{df_2}{f_2^2}.$$

Now for no chromatic aberration $dF=0$, *i.e.* the focal length is the same whatever the colour of the light.

Consequently

$$\frac{df_1}{f_1^2}+\frac{df_2}{f_2^2}=0.$$

But

$$-\frac{df_1}{f_1}=D_1 \quad \text{and} \quad -\frac{df_2}{f_2}=D_2$$

Hence

$$\frac{D_1}{f_1}+\frac{D_2}{f_2}=0$$

is the condition for an achromatic combination. Since both D_1 and D_2 are always positive, the focal lengths must be of opposite sign. Thus one lens is convex while the other is concave, so that the dispersion caused by the convex lens is exactly balanced by the dispersion in the opposite direction caused by the concave lens.

(*b*) Lenses separated by a distance.

If the two lenses are separated by a certain distance they may both be of the same material and may also be of the same type, *i.e.* both convex. The focal length F of such a system has been shown (p. 69) to be

CONVENTION A

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{a}{f_1 f_2}.$$

CONVENTIONS B and C

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{a}{f_1 f_2}.$$

Differentiating, with F, f_1 and f_2 as variables, we have

CONVENTION A

$$-\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} - \frac{a(f_1 df_2 + f_2 df_1)}{f_1^2 f_2^2}$$

or $$-\frac{dF}{F^2} = \frac{D_1}{f_1} + \frac{D_2}{f_2} + \frac{a}{f_1 f_2}(D_1 + D_2)$$

CONVENTIONS B and C

$$-\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} + \frac{a(f_1 df_2 + f_2 df_1)}{f_1^2 f_2^2}$$

or $$-\frac{dF}{F^2} = \frac{D_1}{f_1} + \frac{D_2}{f_2} - \frac{a}{f_1 f_2}(D_1 + D_2)$$

where D_1 and D_2 are the respective dispersive powers. But for lenses of the same material

$$D_1 = D_2.$$

Also the condition for achromatism is

$$dF = 0.$$

CONVENTION A

Hence $$\frac{1}{f_1} + \frac{1}{f_2} = -\frac{2a}{f_1 f_2},$$

or $$a = -\frac{f_1 + f_2}{\quad}$$

CONVENTIONS B and C

Hence $$\frac{1}{f_1} + \frac{1}{f_2} = \frac{2a}{f_1 f_2},$$

or $$a = \frac{f_1 + f_2}{\quad}$$

is the condition for no chromatic aberration. So chromatic aberration may be eliminated by using two convex lenses at a distance apart equal to the mean focal length. This fact is made use of in most optical instruments where an eyepiece consisting of two lenses is used instead of a single lens.

EXAMPLES ON CHAPTER IV

1. What is meant by a caustic curve (or caustic surface) in geometrical optics? Discuss from this point of view the image of a point object formed by refraction at a plane surface separating a dense from a less dense medium, as well as the images formed when parallel light is incident axially (*a*) on a spherical, (*b*) on a parabolic concave mirror. (Camb. Schol.)

2. Discuss the defects of a simple lens when used for the formation of images. (Camb. Schol.)

3. Discuss briefly two of the principal defects to be found in the image produced by a single lens. (Camb. Schol.)

4. Give a diagram, showing how a ray of light is dispersed by a prism.

The image of a very small source of white light is thrown on to a screen by a convex lens. If the screen is brought nearer the lens, the spot of light is enlarged to a round patch and its edge is coloured. How do you explain this? (Lond. Inter.)

5. Explain carefully why chromatic aberration is not apparent when a simple convex lens is used as a magnifying glass.

A narrow slit, illuminated by white light, is viewed (*a*) through a prism, (*b*) through a slab of glass with parallel faces. Describe and explain any differences you would observe. (Lond. H.S.C.)

6. Explain the dispersion produced by a simple lens, and show how the defect may be corrected.

Why is such correction unnecessary in the case of a simple convex lens used as a magnifying glass held close to the eye? (O. & C.)

7. Define dispersive power. Find the relation between the focal lengths of two thin lenses in contact in terms of the dispersive powers of the materials of which they are made, in order that the combination may be achromatic for two colours. (Lond. H.S.C.)

8. Explain the terms dispersion, dispersive power.

What is an achromatic combination of lenses? Describe its construction and explain its action. (Lond. H.S.C., Subsidiary.)

9. Find the condition for no chromatic aberration of a lens system, the lenses being of the same material and being separated by a distance d cm.

Discuss the advantages of an eyepiece, consisting of two lenses, over a single lens for use as the eyepiece of an optical instrument.

10. A convex lens, of focal length 30 cm., achromatic for the two hydrogen lines C and F, is to be constructed from two lenses—one

of crown glass and the other of flint glass. Find the focal lengths of these lenses if

$$\mu_C = 1{\cdot}5145 \quad \mu_F = 1{\cdot}5230 \text{ for crown glass,}$$
$$\mu_C = 1{\cdot}6444 \quad \mu_F = 1{\cdot}6637 \text{ for flint glass.}$$

11. Explain how it is possible to construct achromatic lenses. Why did Newton consider it to be impossible?

An achromatic telescope objective of 100 cm. focal length is to be formed with two lenses made of glass specified below. Find the focal length of each of these lenses, stating whether it is divergent or convergent.

	μ red	μ blue
Glass A . . .	1·5155	1·5245
Glass B . . .	1·641	1·659

(N.)

CHAPTER V

THICK LENSES

WHEN using a thin lens distances are always measured from the centre of lens. The focal length is the distance from the centre of the lens to either focus. But if the lens is a thick one, or is made up of several lenses, the distance from its centre to one focus is different from the distance to the other focus, and neither distance equals the focal length of the lens or lens system. Also if an attempt is made to determine the focal length by any of the simpler methods, involving measurement of such quantities as u and v, we do not know from which point of the lens we should measure these distances.

However, it can be shown that there are two points or planes from which distances may be measured in all cases, and the simple formula for lenses may be applied as usual. These points are called principal points, and are defined as conjugate points on the axis of the lens for which the magnification is unity. On account of this latter fact these points are sometimes known as unit points. The planes which are perpendicular to the optic axis and which contain the principal points are called principal planes. A similar term, unit plane, is used in connection with unit points. Considering first the simple case for a thin lens, the condition for unit magnification is

USING CONVENTION A	USING CONVENTION B	USING CONVENTION C
$\frac{v}{u}=1$, or $v=u$.	$\frac{v}{u}=1$, or $v=u$.	$-\frac{v}{u}=1$, or $v=-u$.
Also $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.	Also $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.	Also $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.
The only solution is $v=u=0$.	The only solution is $v=u=0$.	The only solution is $v=u=0$.

Thus the principal points H_1, H_2 are both at the centre of the lens, and the action of the thin lens may be represented by the diagram, in which F_1 is the first principal focus, and F_2 the second, OK and IL the object and image respectively, the planes through H_1 and H_2 being the principal planes.

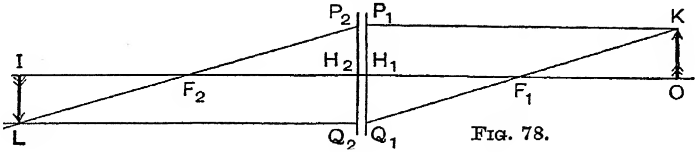


FIG. 78.

It will be seen that a ray from K is taken parallel to the optic axis, meets the first principal plane at P_1, the second at P_2 at the same height, and then passes through F_2. Another ray from K passes straight through F_1, and then becomes parallel to the optic axis after meeting the first principal plane at Q_1.

In the case of the thin concave lens the construction is similar, but now the focal points are reversed and the rays

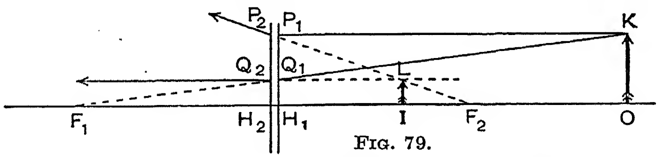


FIG. 79.

only appear to pass through these points. The construction is carried out in exactly the same manner as before.

We may now consider the positions of the principal and focal points of a thick lens. Here the principal points are separated, but the graphic method for finding the position

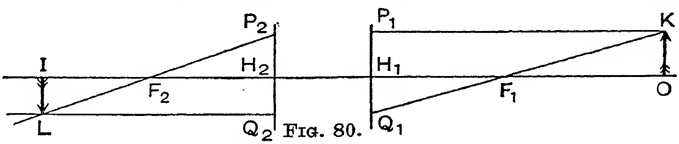


FIG. 80.

of the object may be carried out as before. Fig. 80 shows the construction for a thick bi-convex lens with unit planes at H_1 and H_2. Thus if u is measured from the

first principal plane, and v from the second, the general equation for lenses is still applicable.

Nodal Points.—So far, the assumption has been made that the lens is in one medium, but if this is not the case and the medium on one side of the lens is different from that on the other, it is necessary for us to consider two more points on the axis. These are termed nodal points, and are such that a ray passing through one emerges in a parallel direction through the other. For a thin lens these points are obviously coincident at the centre of the lens. Let C and D be the centres of curvature of the faces of a thick lens in a uniform medium, and let the lines CX, DY be parallel. Then the tangents at X and Y must be parallel, and thus a ray incident at Y will proceed in a parallel direction on emerging, provided it passes through X. The path of the ray may be seen to be ZYXO. If the incident and emergent rays are produced forwards and backwards respectively, they cut the axis in N_1 and N_2, the nodal points.

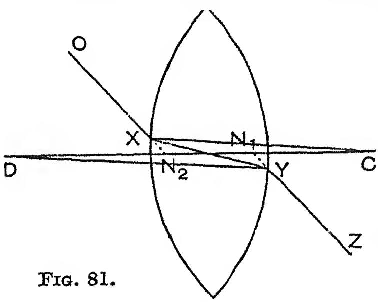


FIG. 81.

It will now be shown that the nodal points coincide with the principal points for a thick lens in a uniform medium, but not if the lens is separating two different media.

Let F_1B be an object in the first focal plane of a lens,

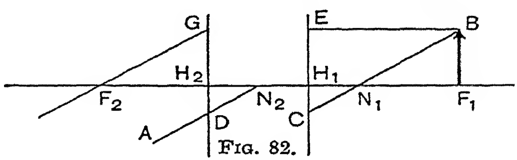


FIG. 82.

the principal and nodal points being denoted by H_1, H_2, and N_1, N_2 respectively. Then a ray BN_1 emerges from the lens in the parallel direction N_2A.

Let the incident and emergent rays cut the principal planes at C and D respectively. Then since principal planes are planes of unit magnification $CH_1=DH_2$, and since BN_1 and DN_2 are parallel, $N_1H_1=N_2H_2$ or $N_1N_2=H_1H_2$. Thus the distance between the nodal points equals the distance between the principal points.

Now let BE be a ray from B parallel to the axis and cutting the first principal plane at E. The emergent ray leaves the second principal plane at an equal distance from the axis and passes along GF_2. Since B is in the focal plane, all rays from it must be parallel after passing through the lens. Thus GF_2 is parallel to DA and also to BN_1. Consequently the triangles BF_1N_1 and GH_2F_2 are congruent, and $F_1N_1=H_2F_2$. Thus

$$H_1N_1=H_1F_1-N_1F_1=H_1F_1+F_1N_1=H_1F_1+H_2F_2=f_1+f_2.$$

If the lens is in a material of uniform density the focal lengths f_1, f_2 are equal but opposite in sign, and so H_1N_1 is zero, or the nodal points coincide with the principal points. But if the lens separates two media of different refractive indices, f_1 and f_2 will not be equal, so that H_1N_1 will not be zero.

The principal points, the nodal points and the focal points of a lens, or system of lenses, are called the cardinal points. In order to determine these points for a lens system the focal points are obtained, and then the focal length found by methods to be outlined. Thus the positions of the nodal and principal points are found. The simplest method for finding the focal points is to place a plane mirror behind the lens system so that the mirror is perpendicular to the axis of the lenses, and an object placed in front of the system is made to coincide with its image. The object is then at one of the foci, and the second focus may be found similarly. Methods for the determination of the focal length of thick lenses will now be considered.

1. *Magnification Method.*—If each term in the lens equation is multiplied by v the equation becomes

CONVENTION A	CONVENTION B	CONVENTION C
$1-\frac{v}{u}=\frac{v}{f}$,	$1-\frac{v}{u}=\frac{v}{f}$,	$1+\frac{v}{u}=\frac{v}{f}$,
or since $m=\frac{v}{u}$,	or since $m=\frac{v}{u}$,	or since $m=-\frac{v}{u}$
$1-m=\frac{v}{f}$	$1-m=\frac{v}{f}$	$1-m=\frac{v}{f}$

where m is the magnification.

In this equation v and m can be varied so that by differentiation we obtain

$$-dm=\frac{dv}{f}$$

or

$$f=-\frac{dv}{dm}.$$

Accordingly f is obtained by measuring the change in v—the actual value not being required—and by finding also the consequent change in m.

In the experiment the object is usually an illuminated glass scale, and the image is formed on a ground glass screen placed on the opposite side of the lens system. The distance of this screen from some fixed point, such as the nearest face of the lens system, is measured, and the relative size of the image to object is found. A series of such readings is obtained, and then a graph is plotted between the distance of the image from the fixed point and the numerical magnification. The graph is a straight line, the slope of which gives a numerical value for f.

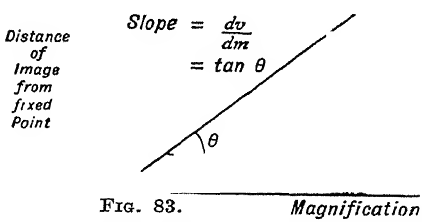


FIG. 83.

2. *Displacement Method.*—If the object and the screen on which the image is formed are at a distance apart greater than four times the focal length (numerically), there are two positions of the lens for which clear images are formed, provided the lens or system acts as a converging lens. Then from measurements of the distance between the two positions of the lens the focal length

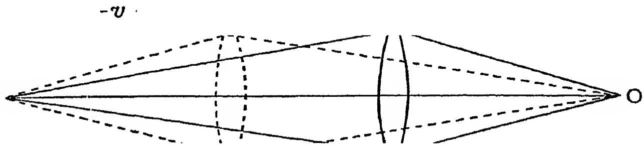


FIG. 84.

may be determined. Let the distance between object and screen be x and that between the two positions of the lens be d. Also let u, u' and v, v' denote the distances of object and image from the respective principal planes.

It is seen from the diagram that

CONVENTION A	CONVENTION B	CONVENTION C
$u=-v',\ u'=-v.$	$u=-v',\ u'=-v.$	$u=v',\ u'=v.$
Now $x=u-v$	Now $x=v-u$	Now $x=u+v$
and $d=u-u'$ $=u+v.$	and $d=v'-v$ $=-(u+v).$	and $d=u-u'$ $=u-v.$
Thus $u=\frac{x+d}{2}$	Thus $u=-\frac{x+d}{2}$	Thus $u=\frac{x+d}{2}$
and $v=\frac{d-x}{2}.$	and $v=\frac{x-d}{\ }$	and $v=\frac{x-d}{\ }$
But $\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$	But $\frac{1}{v}-\frac{1}{u}=\frac{1}{f},$	But $\frac{1}{v}+\frac{1}{u}=\frac{1}{f},$
$\therefore\ \frac{2}{d-x}-\frac{2}{x+d}=\frac{\ }{f},$	$\frac{\ }{-d}+\frac{\ }{x+d}=\frac{\ }{f},$	$\therefore\ \frac{\ }{x-d}+\frac{\ }{x+d}=\frac{\ }{f},$

CONVENTION A	CONVENTION B	CONVENTION C
$\therefore \dfrac{4x}{d^2 - x^2} = \dfrac{1}{f}$,	$\therefore \dfrac{4x}{x^2 - d^2} = \dfrac{1}{f}$,	$\therefore \dfrac{4x}{x^2 - d^2} = \dfrac{1}{f}$,
or $f = \dfrac{d^2 - x^2}{4x}$.	or $f = \dfrac{x^2 - d^2}{4x}$.	or $f = \dfrac{x^2 - d^2}{4x}$.

As a result only the distances d and x need be measured in order to give a value for f.

3. *Minimum Distance Method.*—It has already been shown that for a thin lens the minimum distance between object and image, for the image to be real, is four times the focal length (numerically). In the case of a thick lens this has to be modified owing to the distance between the principal planes, and if this distance is t then the minimum distance between object and image is $4f + t$.

Now the distance between the two principal foci of a thick lens is $2f + t$. Hence if the positions of these foci are determined by aid of a plane mirror, as already explained, this distance $(2f + t)$ may be measured. In addition the object may be placed so that it is at a minimum distance from the real image which is formed, as described for thin lenses. Thus the distance $(4f + t)$ may be measured. Knowing $(4f + t)$ and $(2f + t)$, then by subtraction a value for f may be obtained.

4. *By Use of Newton's Formula.*—It is not possible to measure distances from the principal planes, but in this method all distances are measured from the focal points.

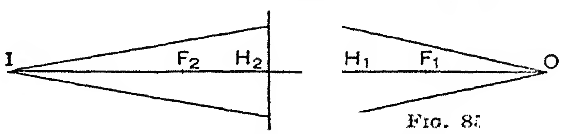


FIG. 85

The experiment is exactly the same as the u and v method, and may be applied to a thick lens or lens system whether that be convex or concave. Instead of measuring u and v, the distances of the object from the first focal point and of the image from the second focal point are found. Let these distances be p and q respectively.

Then for a convex system

CONVENTION **A**

$u = H_1O = p - f$,

and $v = H_2I = f + q$.

Substituting in

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{f+q} - \frac{1}{p-f} = \frac{1}{f},$$

or $pf - f^2 - f^2 - qf$
$= pf - qf - f^2 + pq$,

whence $f^2 = -pq$.

Since p $(=F_1O)$ is positive and q $(=F_2I)$ is negative the right-hand side of the equation is positive, and a real value results for f.

CONVENTION **B**

$u = H_1O = p - f$,

and $v = H_2I = f + q$.

Substituting in

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{f+q} - \frac{1}{p-f} = \frac{1}{f},$$

or $pf - f^2 - f^2 - qf$
$= pf - qf - f^2 + pq$,

whence $f^2 = -pq$.

Since p $(=F_1O)$ is negative and q $(=F_2I)$ is positive the right-hand side of the equation is positive, and a real value results for f.

CONVENTION **C**

$u = H_1O = p + f$,

and $v = H_2I = f + q$.

Substituting in

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{f+q} + \frac{1}{p+f} = \frac{1}{f},$$

or $pf + f^2 + f^2 + qf$
$= pf + qf + f^2 + pq$,

whence $f^2 = pq$.

Since p $(=F_1O)$ and q $(=F_2I)$ are both distances along which the light actually travels they are both positive, so that a real value results for f.

5. *By Measurement of the Magnification and Distance between Object and Image.*—Taking m as the magnification and x as the distance between object and image, we have

CONVENTION **A**

$$1 - m = \frac{v}{f} \quad . \quad . \quad . \quad (1)$$

and

$$x = u - v.$$

Thus

$$mx = \frac{v}{u}(u - v)$$

CONVENTION **B**

$$1 - m = \frac{v}{f} \quad . \quad . \quad . \quad (1)$$

and

$$x = v - u.$$

Thus

$$mx = \frac{v}{u}(v - u)$$

CONVENTION **C**

$$1 - m = \frac{v}{f} \quad . \quad . \quad . \quad (1)$$

and

$$x = u + v.$$

Thus

$$mx = -\frac{v}{u}(u + v)$$

CONVENTION **A**	CONVENTION **B**	CONVENTION **C**
or	or	or
$mx=v(1-m)$ . . . (2)	$mx=v(m-1)$ . . . (2)	$mx=v(m-1)$ . . . (2)
From (1) and (2) we have	From (1) and (2) we have	From (1) and (2) we have
$f=\frac{mx}{(1-m)^2}$.	$f=-\frac{mx}{(1-m)^2}$.	$f=-\frac{mx}{(1-m)^2}$

Consequently only m and x need be determined to yield a value for f.

An illuminated scale is used for object, and the image is formed on a ground-glass screen. The size of the image is found and the distance between the scale and the screen is measured. It should be noted that this method is only applicable when the image is a real one, and consequently the magnification is negative, showing that the image is always inverted. If several readings are taken and a graph of mx against $(1-m)^2$ is drawn, this graph is a straight line passing through the origin, and the slope gives a value for f. The slope is negative since m is a negative quantity.

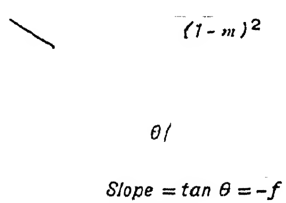


FIG. 86.

When the focal length of the lens or system has been determined, the positions of the principal points may be found by measuring back from the foci. It is advisable to make

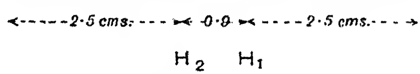


FIG. 87.

a diagram representing the positions of cardinal points, as in the example, so that the action of the system may be seen clearly.

EXAMPLES ON CHAPTER V

1. Show how to find the position of the image of a given object formed by two thin, coaxial, convex lenses of focal lengths f_1 and f_2, and d cm. apart. What advantages result from the use of two lenses instead of one lens in the eyepiece of a telescope?

(Lond. Inter.)

2. What are the cardinal points of a lens system?

Two thin lenses, with axes coincident, are placed 4 cm. apart, and their foci are found to be on opposite sides of the system and 26 cm. apart. One focus is 12 cm. from the nearer lens. If the focal length of the system is 15 cm., find the positions of the principal points.

3. A convex lens and a concave lens have their axes coincident, and are 6 cm. apart. Determine the positions of the foci if the focal lengths are 16 cm. and 20 cm. respectively.

4. An illuminated scale is placed on one side of a lens system and an image is formed on a screen on the opposite side of the lenses. In this position the magnification is 0·8, and the screen is 15 cm. from a fixed point on the lens system. The scale is moved and another position is found for the screen so that the image is distinct. The magnification is now 3·5, and the screen is 30 cm. from the fixed point. Find the focal length of the system.

5. Two similar plano-convex lenses are placed in contact. The radius of curvature of the curved face of each is 20 cm., and the refractive index of each lens is 1·5. Find the focal length of the system and determine this value when the lenses are separated by 5 cm. and then by 10 cm.

6. Describe any method by which you could determine the positions of the principal and focal points of a thick lens.

7. Find the positions of the cardinal points of a sphere of glass ($\mu = 1{\cdot}5$), of radius 3 cm.

8. A double-convex lens has a thickness of 2 cm. along its axis, and has spherical surfaces each of 10 cm. radius.

Obtain the position of an image due to an object on the axis and 30 cm. from the nearest surface ($\mu = 1{\cdot}50$). (Camb. Schol.)

CHAPTER VI

THE EYE AND ITS DEFECTS

The eye is nearly spherical in shape, the skin of this sphere being opaque except for a small circular section. This skin S is called the sclerotic, while the transparent part is termed the cornea C. The inner surface of the sclerotic at the back of the eye is covered with nerve tissues which are sensitive to light. This is called the retina, R, and light from infinity is focussed on this. The greater part of the eye is filled with a liquid called the vitreous humour, V, the refractive index of which is 1·34. Behind the cornea is a chamber called the anterior chamber, A,

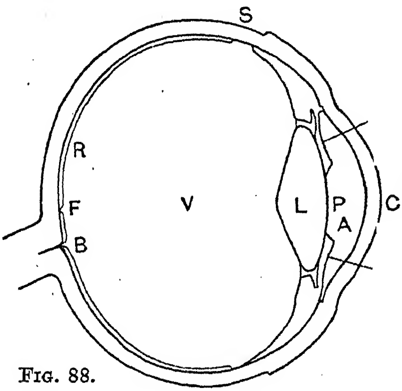


Fig. 88.

which contains the aqueous humour ($\mu = 1{\cdot}34$). Separating this from the vitreous humour, which is contained in the posterior chamber, is the crystalline lens L. This lens consists of a number of layers which increase in density as we pass to the centre, so that the refractive index is different for each layer. The average refractive index is between 1·43 and 1·44. The central section of this lens is called the pupil, P, while acting as a stop or diaphragm just in front of lens is the iris, I, which is coloured. This may open or close to some extent in order to regulate the amount of light which enters the eye. Connecting the lens to the inner surface of the sclerotic is an annular tissue containing the ciliary muscle. This muscle can alter the curvature

of the crystalline lens, and by so doing can change the focal length so that objects at infinity or near to the eye may be focussed. This change in focal length is called *accommodation*. Two points on the retina which are important are the fovea centralis, F, a small hollow in the surface where the sensitivity is a maximum, and the blind spot, B, where the optic nerve enters the eye.

The eye thus bears a resemblance to the camera in that it can focus objects either near to, or at a distance. With the normal eye the nearest point of distinct vision is about 10 inches away, although by straining the eye it is possible to see objects much nearer to the eye than this. The far point or the furthest point of distinct vision is at an infinite distance away.

Defective Eyesight.—It has been pointed out that in the normal eye parallel light is brought to a focus on the retina when the eye is at rest. Two of the most common defects of vision are due to the light coming to a focus either in front of or behind the retina.

1. *Myopia.*—The myopic eye is generally longer than the normal eye, so that parallel light is brought to a focus in front of the retina. Thus the far point is not at infinity, but is at some finite distance from the eye. The near point is usually nearer to the eye than for the normal eye.

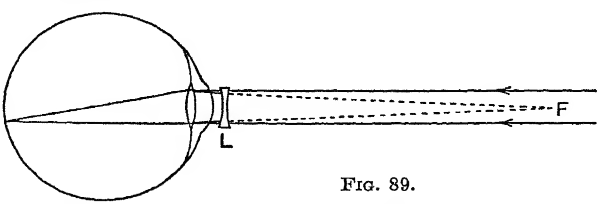


Fig. 89.

In order to correct for myopia, parallel light must be made to appear to come from F, which is the far point for the myopic eye. Thus a concave lens, L, of focal length FL, placed in front of the eye will make distant objects appear distinct. At the same time the lens will cause the near

point to be further from the eye, so that this point is approximately at the normal distance away.

2. *Hypermetropia.*—This defect is due to the eye not being sufficiently long, so that parallel rays would be brought to a focus behind the retina provided the eye was at rest. By the power of accommodation these rays may be focussed, as also may rays from objects closer to the eye. But this means that the eye is never at rest when looking at any object. Also the near point is farther away than for the normal eye; so this defect is often called

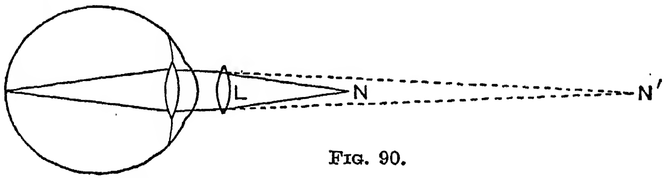


FIG. 90.

long-sight. Correction is made by placing a convex lens in front of the eye, the focal length, f, of the lens being found from

USING CONVENTION A	USING CONVENTION B	USING CONVENTION C
$\frac{1}{d'}-\frac{1}{d}=\frac{1}{f}$,	$\frac{1}{d}-\frac{1}{d'}=\frac{1}{f}$,	$\frac{1}{d}-\frac{1}{d'}=\frac{1}{f}$,

where d = LN = distance of normal near point from the lens or from the eye, and d' = LN′ = distance of near point for hypermetropic eye from lens.

By this means rays from the point N pass through the lens and appear to the eye to have come from N′. The distances LN and LN′ being found by experiment, the focal length of the convex lens may be calculated. Thus a hypermetropic eye which cannot see objects closer than at N′ can, by aid of a convex lens, view objects placed at N (*i.e.* at 10 inches away).

3. *Presbyopia.*—With increasing age the ciliary muscles of the eye deteriorate so that the accommodation power becomes less. The presbyopic eye can see distant objects clearly, but, owing to loss of accommodation power, nearer

objects cannot be observed clearly. In this case the crystalline lens of the eye does not bring rays from objects near to the eye to a focus on the retina, and a convex lens must be placed in front of the eye in order to make the rays more convergent. If the accommodation power has decreased almost to zero, then the object to be viewed must be placed at the focus of the lens. Obviously for viewing distant objects no lens is needed. If myopia accompanies presbyopia a concave lens is needed to correct for the former and a convex lens for the latter. These lenses are often combined one above the other, the resulting lens being called bifocal.

4. *Astigmatism.*—In the astigmatic eye the focal length of the crystalline lens is different in different planes, so that parts of the object are more clearly focussed than others. This is due to the lack of symmetry of the surface of the cornea. If a series of lines equally spaced in the form of a semicircle are viewed by the astigmatic eye some appear more clearly defined than others. In order to correct for this a cylindrical lens is required, so that along the plane perpendicular to the axis of the cylinder the lens acts as an ordinary convex or concave lens, as the case may be. Thus in this plane the eye is aided so that rays are brought to a focus on the retina. If the eye suffers from no other defect the other face of the lens is plane, and the lens is termed astigmatic. Often myopia or hypermetropia is associated with astigmatism, so the lens must be a sphero-cylindrical one, the spherical face being concave or convex.

Many of the modern lenses for spectacles are of the meniscus type. Such lenses possess the advantage over the ordinary type that no distortion is produced when the eye looks at an object not on the axis of the lens. A meniscus lens with one face spherical and the other cylindrical is called a toric lens and is used to correct astigmatism.

Power of a Lens.—Opticians are always concerned with

the power of the lens, not with its focal length. The power of a lens is the reciprocal of the focal length, a lens of short focal length being spoken of as a strong lens. The unit of power is the *dioptre*, this being the power of a convex lens of focal length one metre. For a convex lens the power is positive, while for a concave lens the power is negative. Thus in order to find the power of a lens the reciprocal of the focal length in metres is obtained, *e.g.* the power of a convex lens of focal length 20 centimetres is 5 dioptres; for a concave lens of focal length 8 centimetres the power is $-12{\cdot}5$ dioptres.

The Stereoscope.—It is always possible to estimate the distance away of an object when it is observed simultaneously by the two eyes, provided the distance is not too great. This is only possible because the eyes view the object from slightly different positions. If a photograph is examined, however, it is impossible to judge distances of objects represented on this, since it reproduces the view from one standpoint only. By use of the stereoscope it is possible, when observing photographs, to see the scene actually as it would appear to an observer. Two photographs of the one scene are taken at the same time from two slightly different points. These two photographs are placed side by side in the stereoscope, and are observed through achromatic convex lenses. One photograph is viewed by the right eye and the other by the left. The axes of the two lenses are not quite parallel, but are so arranged that the images of the photographs overlap, so that there appears to be only one photograph. The various parts of this image appear in relief just as they would if the actual scene was viewed by an observer.

Stereoscopy has been applied in the examination of tracks of α particles, which are emitted by radio-active bodies. Two photographs are taken from slightly different angles, and by examination in the stereoscope the actual tracks seem to stand out compared with other parts of the photographs.

In astronomy the stereoscope is of great use. The relative motions of celestial bodies may be determined by viewing in a stereoscope photographs of some part of the sky taken at intervals. By taking photographs of the moon from different positions and examining these stereoscopically it has been found possible to measure the depths of craters on the moon. In a similar manner surveys of land surfaces can be made by viewing, in a stereoscope, pairs of aerial photographs. Such photographs are taken from slightly different viewpoints so that the appearance of solidarity results when a stereoscopic examination is made, and contour maps may be drawn up from a series of these photographs.

The same principle of stereoscopic vision has been employed in the design of certain optical instruments, including range-finders. For distances greater than about 500 yards the stereoscopic effect is too small if only the eyes are used. By using two parallel telescopes—one for each eye—the range may be extended, and distant objects made to stand out in relief. In a similar manner two microscopes may be used for viewing small objects. These instruments are dealt with more fully in the next chapter, where other optical instruments are also considered.

EXAMPLES ON CHAPTER VI

1. Draw a diagram showing a section of the human eye, labelling those parts of special interest in the study of its optical behaviour.

An eye from which the lens has been removed may be considered to be filled with a homogeneous medium of refractive index 4/3. If the distance of the cornea from the retina of such an eye is 2·2 cm., and the radius of curvature of the cornea = 8 mm., find the focal length of a spectacle lens which, placed close to the eye, will enable it to focus a distant object on the retina. (N.)

2. Compare and contrast the optical arrangements of the human eye and those of a photographic camera.

A short-sighted person can only see objects distinctly if they lie between 8 cm. and 100 cm. from the eye. What spectacles would he require in order to see a star distinctly? With these spectacles what would be the least distance of distinct vision? (Lond. Inter.)

3. A person whose nearest distance of distinct vision is 15 cm. uses a lens of 5 cm. focal length to magnify a small object. What is the distance away of the object when in focus, and what magnification is obtained? (Lond. Inter.)

4. A convex lens of 2 in. focal length is held 1 in. from the eye by a person with distance of distinct vision of 9 in. so as to look at a small object. Where must the small object be placed? Illustrate your answer by a figure. (Lond. Inter.)

5. Give a brief general account of the eye as an optical instrument. Suppose a short-sighted eye can see an object clearly only when it is placed at a distance not exceeding 8 in., what kind of lens should be used, and of what power, in order that if placed close to the eye it would enable objects that are 48 in. away to be clearly seen? (Lond. Inter.)

6. A person cannot see objects closer than 40 cm. from the eye. Find the power of the lens required to enable him to see objects up to 25 cm. from his eye.

7. A person wearing spectacles looks over them and is able with one eye to see the moon both directly and also through one of the lenses. The image of the moon is found to be displaced downwards through a distance of one moon diameter. Assuming the moon to subtend an angle of 1° at the eye, and the diameter of the spectacle lens to be 3 cm., find the focal length of the lens and explain the defect of vision which it is designed to correct. (Camb. Schol.)

CHAPTER VII

OPTICAL INSTRUMENTS

The Magnifying Glass.—The nearer an object is to the eye the larger is the angle it subtends. In order for the object to be seen clearly it cannot be brought closer than the nearest point of distinct vision without unduly straining the eye. But if a convex lens is placed close to the eye the object may be brought nearer and still be seen clearly. Thus the angle subtended by the object is increased, or, in other words, the object is magnified. A convex lens used in this manner is called a magnifying glass. The

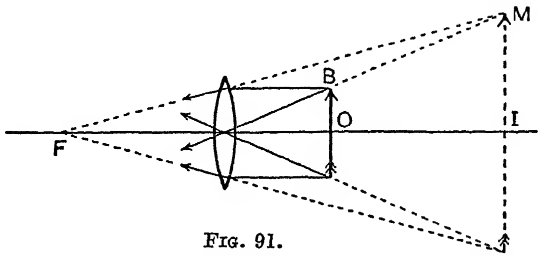


FIG. 91.

object is nearer to the lens than the focus, and a virtual image is obtained. If the image is formed at the nearest point of distinct vision we may replace v by d, the distance of the near point from the eye, in the lens equation. Thus

USING CONVENTION A	USING CONVENTION B	USING CONVENTION C
$\frac{1}{d}-\frac{1}{u}=\frac{1}{f}$	$-\frac{1}{d}-\frac{1}{u}=\frac{1}{f}$	$-\frac{1}{d}+\frac{1}{u}=\frac{1}{f}$
since $v=d$,	since $v=-d$,	since $v=-d$,
$\therefore\ m=\frac{d}{u}$	$\therefore\ m=-\frac{d}{u}$	$\therefore\ m=\frac{d}{u}$
$=1-\frac{d}{f}.$	$=1+\frac{d}{f}.$	$=1+\frac{d}{f}.$

For good magnification f should be small so that the magnification is approximately equal to d/f and is positive.

If a white object is viewed through a simple magnifying glass no chromatic aberration is apparent. This is due to the fact that the separate images formed by the different coloured rays subtend exactly equal angles at the eye, and so completely overlap. Let L be the length of the object, f_b and f_r the focal lengths of the lens for blue and red light respectively, and d_b, d_r the corresponding distances away of the blue and red images. Then the length of the blue image

$$=\frac{d_b}{u}\times L,$$

and length of red image

$$=\frac{d_r}{u}\times L.$$

Each image subtends an angle at the eye, the angle being $\left(\frac{d_b}{u}L\frac{1}{d_b}\right)$ or $\frac{L}{u}$ in the former, and $\left(\frac{d_r}{u}L\frac{1}{d_r}\right)$ or $\frac{L}{u}$ in the latter case.

Thus, provided the lens is close to the eye, the angle subtended is the same in both cases, and there is no chromatic aberration.

Eyepieces.—In practically all optical instruments a lens is used in order to magnify an image formed by another lens. The disadvantages of using only one lens for this purpose are that both chromatic and spherical aberration can occur to such a degree that the accuracy of measurements is impaired. In order to eliminate or reduce these defects two lenses are substituted for the one, and the system is called an eyepiece. Two of the most important eyepieces are due to Ramsden and Huygens respectively.

Ramsden's eyepiece consists of two plano-convex lenses

of equal focal lengths, separated by a distance equal to two-thirds of the numerical focal length of either lens. The convex faces are towards each other so that spherical aberration effects are reduced. The condition for achromatism is not quite obtaine but chromatic aberration is reduced and may be eliminated by using two achromatic lenses.

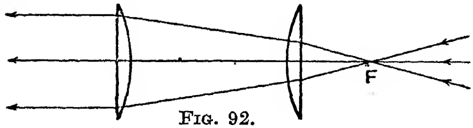


FIG. 92.

The focal length of the combination is $\frac{3f}{4}$, where f is the focal length of either lens, and the focus F lies outside the lenses at a distance of $\frac{1}{4}f$ from the nearer lens. It is at this point that the image formed by the objective falls, and the cross-wires are placed here. This is a great advantage, since for measurement the image can always be brought to a definite point where the wires cross each other. So an image is formed on the cross-wires, and then a magnified image of these is seen on looking through the eyepiece. Ramsden's eyepiece is thus known as a positive eyepiece, since the cross-wires are on the positive side.

Huygens' eyepiece consists of two plano-convex lenses, with the convex faces towards the object, the focal lengths being in the ratio of three to one. The eye lens (*i.e.* the one nearer the eye) is the stronger one, and the distance between the lenses is twice the focal length of this lens.

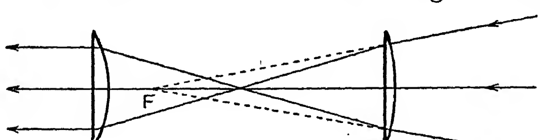


FIG. 93.

This eyepiece was designed to minimise spherical aberration by producing equal deviations at each lens to a ray initially parallel to the axis. Chromatic aberration is

entirely eliminated, since the distance between the lenses is half the numerical sum of the focal lengths.

The focal length of the eyepiece is $\frac{3f}{2}$, where f is the focal length of the eye lens, and the first principal focus, F, lies between the lenses at a distance $\frac{f}{2}$ from the eye lens. The rays from an object, after passing through the objective of the instrument used, converge towards this point, but no image is formed here since the rays first meet the field lens (*i.e.* the lens in the eyepiece nearer to the object) and are consequently deviated. So with this eyepiece no cross-wires can be used, and as the image formed by the objective is on the negative side of the field lens the eyepiece is called a negative eyepiece.

The Telescope.—This instrument is used for viewing objects at a large distance. Although the principle was described by Roger Bacon in 1250, telescopes were not constructed until the beginning of the seventeenth century, when they were used chiefly for astronomy. Telescopes are of two types—refracting and reflecting, the former consisting of a lens and an eyepiece, the latter being made up of a mirror and an eyepiece.

The Astronomical Telescope.—In this instrument an image of a distant object is formed by a convex lens, and the image is observed through an eyepiece acting as a simple magnifying glass. Since the eyepiece acts in a manner similar to a single lens, the telescope is shown in the diagram to consist of two lenses alone, the lens nearer the object being the objective, and the other the eye lens. Three rays from the top and three from the bottom of the object are taken in order to define the position of the image. These rays passing through the objective form a real inverted image, AB, and continuing through the eye lens appear to have come from IM. Thus IM is the final image, virtual and inverted. This image may be

situated anywhere between the near point of the eye and infinity so that it may be seen clearly, but for the eye to be at rest during observations the image should be at

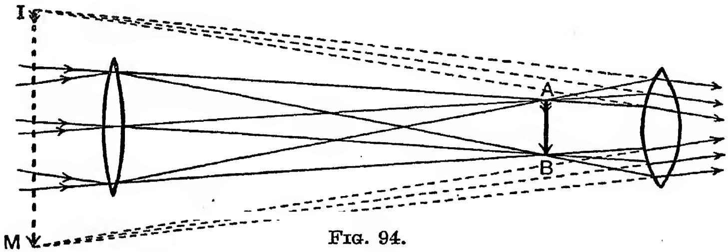


FIG. 94.

infinity. If cross-wires are used in this telescope they are situated at AB in the focal plane of the objective.

Magnifying Power of a Telescope.—A telescope magnifies in that it increases the angle subtended at the eye. Let x be the distance of the object from the objective, and let l be the length of the object. Then since the length of the telescope is negligible compared with the distance x, the angle subtended by the object at the eye $= l/x$.

If the image AB is distant v from the objective and u from the eye lens, while the final image IM is at a distance y from the eye lens, then the magnification at the objective is $-v/x$ and at the eye lens is y/u. So the length of IM is $-l\frac{v}{x}\frac{y}{u}$, and the angle subtended at the eye $= -\frac{lv}{xu}$.

The magnifying power is the ratio of the angle subtended at the eye by the image to the angle subtended by the object. Thus (all distances being counted positive)

$$\text{magnifying power} = -\frac{lv}{xu}\bigg/\frac{l}{x} = -\frac{v}{u}.$$

Now if the object is a large distance away, $v = \mathrm{F}$, the focal length of the objective. Also for the image IM to be seen without eye strain AB must be at the focus of the eye lens, and $u = f$, the focal length of this lens. In this

case the magnifying power $= -\frac{F}{f}$. Consequently to obtain high magnification the telescope should consist of a long focus objective and a short focus eyepiece. The negative sign for the magnification merely indicates that the image is inverted. It should be noticed that this result is a special one, obtained only when the object and the final image are at infinity. But even if these conditions are not realised exactly, the approximation is so small that the value obtained, $-\frac{F}{f}$, may be taken as correct except when the object is brought to within a short distance of the objective. In this case the instrument acts as a compound microscope, and a different value for the magnifying power is obtained.

Determination of the Magnifying Power of a Telescope.—The telescope is focussed on a scale which is placed a large distance away. The scale is observed directly with one eye and through the telescope with the other eye, so that the image is seen superimposed on the scale itself. The ratio of the number of actual scale divisions to the number of divisions seen through the telescope and corresponding to the same distance gives the magnifying power.

The Terrestrial Telescope.—In viewing celestial bodies it does not matter whether the image is upright or inverted. For terrestrial objects the astronomical telescope would be useless, since in this case the image seen must be upright. In order to obtain this a third convex lens is

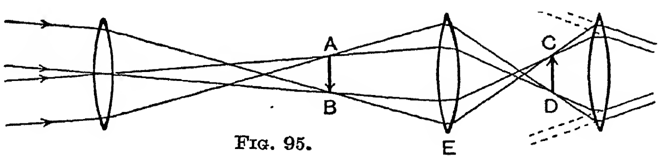


FIG. 95.

needed, this lens being called an erecting lens. A real inverted image AB is formed as usual by the objective,

and the light passes on through the erecting lens E to form a second real image, CD, upright, and the same size as AB.

The image CD is observed through the eye lens in the usual manner, the final image being both virtual and upright. The distance between AB and CD is four times the focal length of the erecting lens, so that this lens should be as strong as possible in order to keep the telescope from being too long. Instead of one lens two plano-convex lenses are often used, so that by producing equal deviations at each surface spherical aberration may be reduced. In the diagram two rays from the top and bottom of the object only are considered for the sake of clarity.

Galileo's Telescope.—Here the eye lens is concave, and the resultant image is virtual and upright. An inverted image would be formed at AB by the convex objective, but before this is formed the converging rays pass through the concave eye lens, giving rise to the virtual, upright image IM. The magnifying power of this telescope may be found as for the astronomical telescope, and is the ratio of the focal length of the objective to the focal length of the eye lens, numerically. In addition to forming an upright image another advantage of Galileo's telescope is that it is shorter than the corresponding astronomical telescope. On account of these two facts, opera glasses

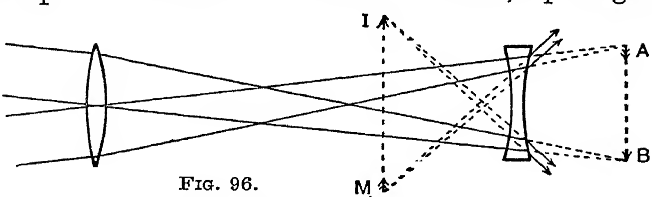


FIG. 96.

may be constructed on the principle of Galileo's telescope. The chief disadvantage of this instrument is that cross-wires cannot be included, so that it is impossible to use it for making measurements. Cross-wires are always situated at the place where the real image is formed, so

that the final image and the magnified image of the cross-wires are seen through the eye lens together, without there being any parallax between them.

In all telescopes the objective or object-glass is larger than the eyepiece. The reason for this can be seen from the diagram, in which it is assumed that the object and the final image are at infinity, and that the lenses have diameters just sufficient to accommodate the beam. Thus

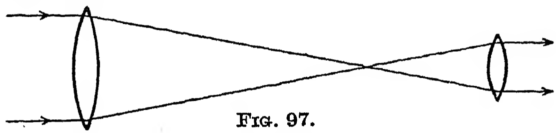


Fig. 97.

$$\frac{\text{diameter of objective}}{\text{diameter of eye lens}} = \frac{\text{focal length of objective}}{\text{focal length of eye lens}}$$
$$= \text{magnifying power (numerically).}$$

As will be seen later, the resolving power of a telescope depends on the size of the objective, so that this lens is made as large as possible, the diameter of the eye lens or eyepiece having a value in accordance with the above relation. Also the larger the objective the greater is the intensity of illumination of the final image.

Mirror Telescopes.—Believing that it was impossible to form a compound lens which was achromatic, Newton constructed a telescope which had as objective a concave mirror, the eyepiece consisting, as usual, of two lenses.

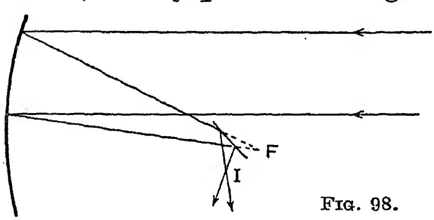


Fig. 98.

Parallel light falling on the objective is brought to a focus at F. Just before the rays form the image here, a small plane mirror is placed in their path at an angle of 45° with the axis of the instrument so that the light is reflected to the side. The image is thus formed at I and is viewed

through the eyepiece. Other mirror telescopes based on the same principles have been designed by Herschel, Gregory and Cassegrain. In the modern mirror telescope the mirror is paraboloidal and not spherical. By this means spherical aberration is completely eliminated, and since the objective is a mirror, there is no chromatic aberration.

Magnifying Power of a Mirror Telescope.—The expression for the magnifying power of a mirror telescope may be found in exactly the same manner as for refracting telescopes. It will be recalled that the expression then obtained was

$$\text{magnifying power} = v/u \text{ (numerically)},$$

where v is the distance of the first (real) image from the objective, and u is the distance of this image from the eye lens.

For a reflecting telescope we have the same expression. Now when the object is a large distance away, $v = \mathrm{F}$, the focal length of the mirror; also, for the final image to be at infinity, $u = f$, the focal length of the eye lens. Consequently the magnifying power is F/f numerically, indicating that a mirror of large radius of curvature is required together with a short focus eye lens.

Comparison of Refracting and Reflecting Telescopes.—The chief advantages of mirror telescopes are:

(1) There is no chromatic aberration.

(2) Spherical aberration may be eliminated by using paraboloidal mirrors.

(3) Mirrors can be manufactured with larger diameters than lenses, and so give a higher resolving power. This is a point of great importance in astronomy when examinations of star clusters and double stars are being carried out.

In refracting telescopes chromatic aberration may be eliminated by employing an achromatic objective, while

spherical aberration can be reduced—but not entirely cut out—by use of stops.

For general work refracting telescopes are considered to be much better than reflecting telescopes, since the latter are less convenient to use and need frequent readjustment. It is only when high resolving power is required that the mirror telescope is superior, so that this type of telescope is not frequently used.

The largest telescope in the world is the 100-inch reflecting telescope at Mount Wilson Observatory. This telescope was used by Michelson in his interference method for determining the angular diameters of stars.

The biggest refracting telescope is the one at Yerkes Observatory. The diameter of its objective is 40 inches, and its focal length is 65 feet.

Prism Glasses.—Although Galileo's telescope is shorter than other telescopes, it is inconveniently long and cumbersome for field glasses or opera glasses, in which two telescopes are mounted so that both eyes view the object simultaneously. In order to shorten the length of a telescope without changing the length of path of the rays, two right-angled prisms are introduced between the objective and the eyepiece. The lenses are both convex, but the image is not inverted as in the ordinary telescope since this inversion is completely balanced by arranging the prisms with their axes at right angles. One prism renders the image erect but does not affect the lateral inversion, while the other prism corrects the lateral inversion and so leaves the final image erect and the right way round. The diagram shows the arrangement of the prisms in half the instrument. The other half is similar so that binocular vision is secured.

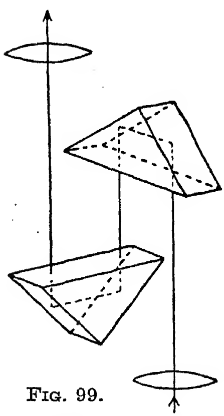


Fig. 99.

The Periscope.—The principle of the telescope is again used in the periscope, where two right-angled prisms are included so that the light may be reflected and its direction changed. The incident light falls on one of the smaller faces of a right-angled prism and is reflected by this prism through the objective O. It then passes down the instrument to an erecting lens, E, before reaching a second right-angled prism which so reflects the light that its direction is parallel to that of the incident beam. Finally the light passes through an eyepiece, P, to the eye. Cross-wires are included, and are placed in the focal plane of the eyepiece.

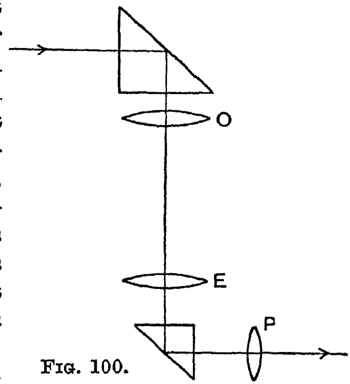


FIG. 100.

Night Glasses.—An object which is so feebly illuminated that it cannot be seen by the naked eye can often be seen if viewed through a telescope. The reason for this is that the parallel beam of light reaching the objective becomes a much narrower beam of light when it emerges from the eyepiece, so that the intensity of illumination is increased since the area illuminated is reduced.

Now, as shown on page 120,

$$\text{magnifying power} = \frac{\text{diameter of objective}}{\text{diameter of eyepiece}},$$

showing that an increase in the magnifying power of a telescope means an increase in the visibility. There is a limit to this, however. If the emergent pencil of light just fills the pupil of the eye of the observer, then any decrease in the diameter of the eyepiece has no effect as regards visibility, since no more light falls on the retina. Thus a limit is reached when the diameter of the eyepiece equals the diameter of the pupil of the eye (about 8 mm.).

Let D represent the diameter of the objective and d the diameter of the eyepiece, the latter not being less than the diameter of the pupil of the eye. Then a beam of diameter D becomes a beam of diameter d, so that the intensity of illumination is increased in the ratio $D^2 : d^2$, since the intensity of illumination depends on the area illuminated.

But $D/d =$ magnifying power.

Thus the intensity of illumination at the eye is proportional to the square of the magnifying power of the telescope, neglecting the small loss of light by reflection and absorption in the instrument.

√*The Compound Microscope.*—A single lens used as a magnifying glass does not give high magnification. By using two lenses it is possible to obtain a very high degree of magnification, so that objects invisible to the naked eye can be seen clearly through the microscope. A real inverted image of an object OB is formed at AC by means

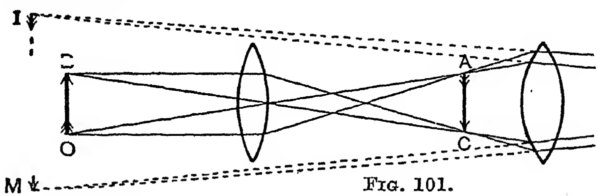


FIG. 101.

of a convex objective. It is essential that this image shall be a real one; therefore, OB must be placed beyond the focus of the objective. At the same time, since high magnification is desired, the distance of OB from the lens must not be large, and so the object should be close to the focus. The image AC is then viewed through an eyepiece, acting as a simple magnifying glass, and a virtual image, IM, is formed. It can be seen that magnification is produced at both the objective and the eyepiece.

If the focal lengths of the objective and eyepiece are F and f respectively, the magnification produced by the eyepiece is

CONVENTION A	CONVENTION B	CONVENTION C
$\left(1-\frac{d}{f}\right)$	$\left(1+\frac{d}{f}\right)$	$\left(1+\frac{d}{f}\right),$

where d is the distance of the near point from the eye. Let the distances of OB and AC from the objective be u and v respectively. Then the magnification at the objective is

$\frac{v}{u},$	$\frac{v}{u},$	$-\frac{v}{u},$

so that the total magnification is

$\left(1-\frac{d}{f}\right)\frac{v}{u}.$	$\left(1+\frac{d}{f}\right)\frac{v}{u}.$	$-\left(1+\frac{d}{f}\right)\frac{v}{u}.$

In order to make this as large as possible, both v/u and d/f must be large. The former can be achieved by making u very small, which means that F should be small since u must be slightly larger than F, while d/f is made large by taking f very small. Thus both the objective and the eyepiece should have short focal lengths.

In using the instrument the eyepiece is first focussed on the cross-wires, and the whole instrument is then moved towards or away from the object until a clear, magnified image is seen coincident with the cross-wires. Often a microscope is used for measuring small distances, and it is therefore mounted so that it can be moved along a scale, the actual position being read off by means of a vernier.

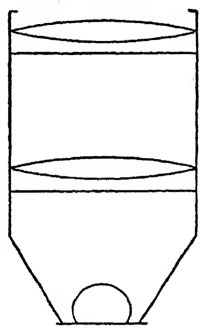


FIG. 102.

In all microscopes an eyepiece is used instead of a single eye lens, while the objective also consists of a number of lenses. The objectives of high power microscopes have (at least) two achromatic lenses separated by a short distance, a third lens, almost a hemisphere, also being included, its plane face being towards the object. An objective of this type has a very small focal length, and

can be placed quite close up to the object, high magnification resulting.

Sometimes it is desirable to use a binocular microscope. A system of prisms, as indicated in the diagram, is then employed so that two beams of light from the object are produced and pass through the two microscopes, one for each eye. It may be wondered why the two parts of this microscope are not parallel as in the binocular telescope. The reason is that a microscopist uses his eyes for observing objects quite close to him, and in such cases the rays to the two eyes do not move in parallel directions. So when an object is viewed through a microscope there is less strain on the eyes if the two parts of the microscope are not parallel.

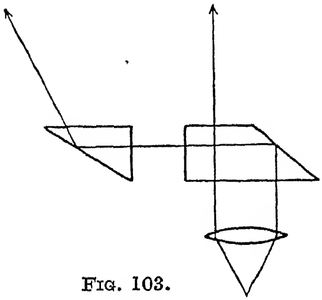


Fig. 103.

Determination of the Magnifying Power of a Microscope.—The microscope is focussed on to a fine scale, divided into, say, hundredths of a millimetre. Between the eye-piece and the eye a thin piece of glass—a cover slip—is placed at about 45° so that it will reflect light into the eye from another scale, this one being divided into centimetres and millimetres. Thus the eye sees the two images superimposed—one being the magnified image formed by the rays passing through the microscope, the other being formed by reflection and having unit magnification. Then if N divisions of the scale viewed through the microscope correspond to n millimetres on the other scale, the magnifying power of the microscope $= 100\frac{n}{N}$, since 100 divisions on the fine scale equal 1 millimetre. There must be no parallax between the two images, so that the centimetre scale must be adjusted in position until the images coincide exactly.

The Sextant.—This instrument is used in order to find the angle subtended by two distant objects. If one of the objects chosen is the horizon at sea, then the angle of elevation of any other object can be found. A plane mirror, M, which may be rotated about an axis perpendicular to the paper is attached to an arm carrying at the end a vernier V. The vernier moves over a circular scale AB.

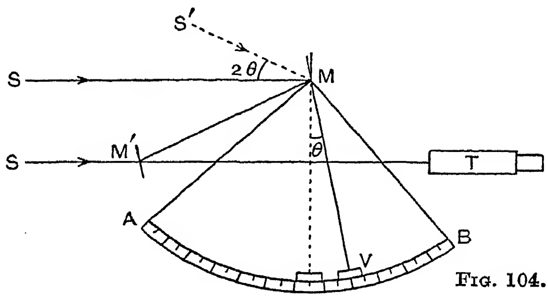


FIG. 104.

A second mirror M′, of which only half is silvered, the other half being clear glass, is fixed so that it is in a plane perpendicular to the paper. The instrument is turned to view a distant object, an image of which is seen through the telescope T, the light having passed through the clear part of M′. The mirror M is rotated so that it is parallel to M′, and so that light from the same object travelling in this direction is reflected from M, falls on M′ and is there reflected into the telescope. Then M is rotated until light from the other distant object coming from the direction S′M is reflected on to M′ and thence into the telescope. Thus the images of the two objects are seen superimposed on looking through the telescope.

Now the angle subtended by the two objects is SMS′, and since a mirror rotates a ray through twice the angle it turns through itself, the angle turned through by M is half SMS′.

Consequently the difference in the positions of V on the

scale AB for the two positions of M gives half the angle subtended by the objects. In order to make the sextant give the actual angle subtended directly, each scale division equals two degrees. Thus, if M is rotated through an angle θ the difference in scale readings is 2θ, giving directly the angle subtended by the objects. If it is required to find the altitude of the sun at sea, the line of the horizon is taken for one object, the sun itself being the other. On land the horizon cannot be used; instead, the image of the sun formed by reflection in a pool of liquid, usually mercury, is used. Then the angle subtended by the sun and its image is twice the altitude required.

The Optical Lantern.—This instrument is designed to project a magnified image of an object on to a screen. The source of light S is usually rather small in size, and in order for the object L, the lantern slide, to be uniformly illuminated, a condenser C—formed of two large plano-

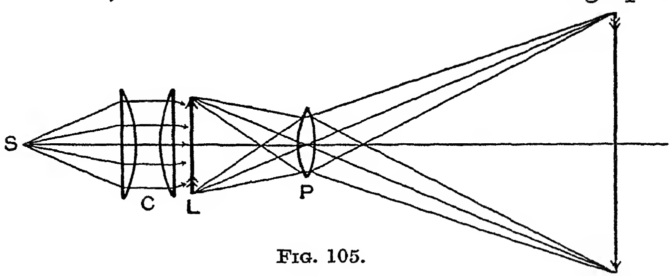


FIG. 105.

convex lenses with the curved faces towards each other—is placed between the source and the slide. The rays from the slide pass through a focussing lens P, which can be moved towards or away from the slide, and then go on to the screen. The image formed is thus an inverted one. Generally the lens P is replaced by two lenses, forming an achromatic combination and also reducing spherical aberration so that a clear image results. Since the condenser forms an image of S near to P, there must be a minimum of spherical aberration at C. For this reason

the two plano-convex lenses are used instead of one lens alone.

The Epidiascope.—In some cases it is desirable to project an enlarged image of an opaque object. For such purposes the episcope is used. In this instrument a powerful source of light S illuminates the object AB, which is usually placed on the horizontal base of the instrument. The light is concentrated by use of condensing lenses C, as in the optical lantern. The light given out by the object is then reflected by a plane mirror M through the projecting lenses, P, which can be adjusted so that an image of the object is formed on a screen.

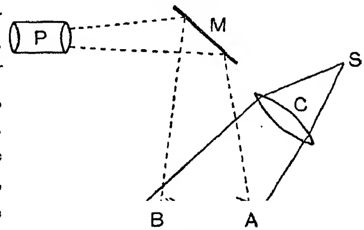


FIG. 106.

Frequently it is convenient to project both transparent objects (diascopic projection) and opaque objects. The instrument employed for this is called an epidiascope. Here the lamp sends out light through condensing lenses to large plane mirrors at the sides of the instrument. These mirrors are tilted at such an angle that the object is brilliantly illuminated. The light given out by the object is then projected as previously described. For projecting slides with this instrument a separate projection tube, with lenses, is used.

In both the episcope and the epidiascope precautions have to be taken against overheating. Since only the light diffused from the object is usefully employed, it is essential to have a powerful source of light. With lamps of high wattage the heat produced is considerable, and in order to overcome this heating effect rotating fans are often employed. In addition, the object is sometimes covered with a plate of heat-resisting glass.

The Cinematograph.—Owing to the persistence of vision, the eye still sees the image of an object for a fraction of

a second, about 1/12th, after the object has been removed. In the cinematograph a series of pictures of moving objects, taken at a rate of at least twelve per second, is projected on to a screen at the same rate, so that the objects appear to move continuously and not in jerks. The film on which the pictures are taken is arranged to move in jerks, so that at each jerk a fresh picture is brought in front of the beam of light and is then projected on to the screen. While the film moves during each jerk a shutter cuts off the light and then moves out of the path of the light while the next picture is stationary.

In order to overcome the flickering effect which is present when the film moves in jerks, a shutterless projector has been designed. In this instrument the film moves continuously, and the light which passes through falls on one of a system of small plane mirrors fixed to the outside of a wheel which rotates continuously. Just as the picture passes in front of the beam of light the transmitted light falls on one of the mirrors and is reflected through the projecting lenses on to the screen. While this picture is moving out of the path of the light, the mirror system is also moving, and none of the reflected light is projected until the next picture is entirely illuminated.

In the talking film the "image" of the sound is formed at the side of the pictures on the film. Light shines through these images, which vary in intensity, so that the intensity of the transmitted light fluctuates. This light falls on to a photo-electric cell, causing electrical impulses to be set up. These are amplified, and the resulting current passed on to loud speakers.

For recording sound the actual systems employed differ in details, but they all act on the principle of producing a varying electrical current as the sounds vary. This current then operates some device, *e.g.* a shutter suspended in a magnetic field, so that the intensity of light which falls on the film is made to vary with the sound.

The Camera.—The simplest type of camera is the fixed

focus or box camera, which consists of a light-proof box with a lens at one end and the sensitive photographic plate at the other. The plate is in the focal plane of the lens, so that sharp images of distant objects may be formed on the plate.

In order to take photographs of objects which may be at any distance away, a camera of variable length is required—the lens-holder being joined to the back of the camera by bellows so that the length of the camera may be varied at will. The quantity of light entering the camera can be regulated by means of an adjustable diaphragm. The diameter of this opening is written as a fraction of the focal length of the lens (*e.g.* $f/11$).

Special care has to be taken in the choice of photographic lenses. The chief defects of lenses—spherical aberration, distortion and chromatic aberration—must be eliminated, or poor photographs will result. In order to overcome chromatic aberration all lenses are made achromatic over the required range of wave-lengths, about 3500 to 5000 A.U.* If the camera is for special use with plates sensitive to infra-red rays, then it is necessary for the lens to be achromatic for the whole range of waves received. The ordinary plate is, however, sensitive only to wave-lengths less than 5000 A.U. Spherical aberration is much reduced if the size of the stop-opening is diminished, but distortion can only be overcome by special arrangements of lenses.

Rectilinear lenses do not exhibit any distortion. Here, two achromatic lenses are placed a certain distance apart with the diaphragm between them. Each lens produces distortion, but these distortions are in opposite directions and so balance each other. Such lenses give good results provided the stop-opening is small, but for general work they have been superseded by the more recent anastigmatic lenses. They still, however, are much used for portraiture, where the image need be sharply defined only over a limited area.

* 1 A.U. $= 10^{-8}$ cm.

Anastigmatic lenses are constructed of a special kind of glass, Jena glass, so that they produce no curvature or distortion even with a large aperture. Such lenses are generally used in pairs, and produce an image in which the outer parts are defined as clearly as the central portions.

Telephotographic lenses are designed so that as large an image as possible shall be formed of a distant object without making the camera so long that it is inconvenient. This system of lenses consists of two achromatic lenses, one being convex and the other concave as illustrated. The distance between the lenses is less than the focal length of the convex lens A, so that a parallel beam of

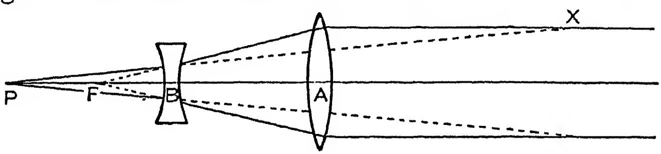


FIG. 107.

light incident on A converges towards F, the focus of A. Before reaching F the light passes through the concave lens B, which reduces the convergency of the beam and brings it to a focus at P. If these rays are produced back until they cut the original beam at X and Y, then it can be seen that the telephotographic system is equivalent to a single convex lens placed at XY, its focus being at P. Thus the telephotographic camera has an effective length which can be made much greater than its actual length.

EXAMPLES ON CHAPTER VII

1. A person whose minimum distance of distinct vision is 20 cm. uses a magnifying glass of 5 cm. focal length held close to the eye. What must be the position of the object examined, and what magnification is obtained?

How would you make a compound microscope with two lenses? Discuss the relations between the position of the object, the distance between the lenses, and the distance of distinct vision. (N.)

2. Explain how a convex lens, held close to the eye, may be used as a simple microscope or magnifying glass.

A man whose least distance of distinct vision is 25 cm. uses a convex lens of 3 cm. focal length as a magnifying glass. Find the magnification he obtains. Illustrate your answer by a diagram approximately to scale. (O. & C.)

3. Explain how a single lens may be used as a microscope, and calculate the magnification obtained.

How would you obtain greater magnification? (Lond. Inter.)

4. Discuss the factors which determine the magnifying power of (*a*) a magnifying glass, (*b*) a microscope. (Ox. Schol.)

5. Explain, with the aid of a diagram, the formation of an image by means of a compound microscope comprising two single lenses. If the observer sees a distinct image at a distance of 25 cm., find the position at which the object must be placed, the focal lengths of the lenses being 5 cm. and 1 cm., and the distance between them being 20 cm. Calculate also the magnifying power. (Lond. Inter.)

6. The object-glass of a microscope has a focal length of 1 in., and the eyepiece a focal length of $1\frac{1}{3}$ in. The lenses are fixed 4 in. apart and focussed on an object so as to form a virtual image 10 in. from the eyepiece. Calculate the magnifying power, and make a diagram showing the passage of two rays, coming from a point on the object not on the axis, through the microscope.

(Lond. Inter.)

7. The objective and the eyepiece of a microscope have focal lengths of 2 cm. and 4 cm. respectively, and are placed 15 cm. apart. If the final image is situated 25 cm. from the eyepiece, how far must the object be placed in front of the objective?

8. What kind of lenses would you choose for making a compound microscope? How does the magnification depend on the distance away of the nearest point of distinct vision from the eye?

9. A microscope is used by a person with normal vision, and then by a short-sighted person. What change in the position of the eyepiece must be made, and how will this affect the magnification?

10. A microscope is set up in the laboratory by placing an object-glass, of 1·5 in. focal length, 12 in. from an eye lens of 2 in. focal length, and setting the object so that the final image, seen by the observer, is 10 in. from the eye lens. Compare the angle subtended by this image at the eye lens with that which the object subtends at a point 10 in. away.

Point out reasons why the optical arrangement of a real microscope is more complicated than that indicated above. (N.)

11. Describe a compound microscope. How may its magnifying power be measured experimentally? (O. & C.)

12. Draw diagrams to show the course of several rays of light through a telescope, starting from a point not on the axis: (*a*) when the eyepiece is a converging lens, (*b*) when the eyepiece is a diverging lens.

An astronomical telescope is arranged for viewing the moon. How will the appearance of the moon as seen through the telescope be affected if the lower half of the object-glass is covered with an opaque screen? (Lond. Inter.)

13. Explain what is meant by the magnifying power of an optical instrument.

Describe the construction of a simple telescope containing two single lenses. When is the telescope said to be in normal adjustment? What is the eye-ring, and where is it situated? Find an expression for the magnifying power of the telescope (*a*) when it is in normal adjustment; (*b*) when the eye is at the eye-ring and the final image is at the least distance of distinct vision. (Lond. Inter.)

14. Describe an astronomical telescope, and draw a diagram illustrating the passage through the system of a pencil of light from a non-axial point on the object to the eye.

How would you determine experimentally the magnifying power of a telescope? (Lond. H.S.C.)

15. Draw a diagram illustrating clearly the passage of rays through an astronomical telescope.

The focal lengths of objective and eyepiece of such a telescope are 12 in. and 1 in. respectively. The telescope is focussed on a scale 6 ft. from the objective, the final image being formed 12 in. from the eye of the observer. Calculate the length of the telescope and the magnification produced by it. (Lond. H.S.C.)

16. An astronomical refracting telescope is adjusted to give a real image of the sun upon a screen. Draw a diagram showing the path of a pencil of rays through the telescope to a point on the boundary of the image.

If the focal lengths of the object-glass and the eye lens are 100 cm. and 2·5 cm. respectively, and the image of the sun, formed on a screen placed 30 cm. from the eye lens, is 9·6 cm. in diameter, find the angle which the sun subtends at the centre of the object-glass. (N.)

17. Describe, and explain the action of, a simple astronomical telescope.

Show how the magnifying power of the instrument can be calculated. (O. & C.)

18. A Galilean telescope has an object-glass of 12 cm. focal

length and an eye lens of 5 cm. focal length. It is focussed on a distant object so that the final image seen by the eye appears to be situated at a distance of 30 cm. from the eye lens. Determine the angular magnification obtained, and draw a ray diagram.

What are the advantages of prism binoculars as compared with field glasses of the Galilean type? (N.)

19. Describe, with the aid of a diagram, the construction and mode of action of a Galilean telescope. What advantages has this system over that of the astronomical telescope?

The objective and eyepiece of a Galilean telescope have focal lengths of 25 cm. and 5 cm. respectively. Calculate the angular magnification produced when the system is used to view an object at a distance of 4 metres from the objective, the final image being formed at the minimum distance of distinct vision (25 cm.) from the eyepiece. (O. & C.)

20. Describe the optical system of a reflecting telescope, giving a diagram showing the path of three selected rays from the same point not on the axis. Is the image upright or inverted in the case you describe? (O. & C.)

21. Describe the construction of some form of mirror telescope. Discuss the advantages and disadvantages of reflecting telescopes as compared with refracting telescopes.

22. Describe the construction of the sextant and the periscope. Illustrate your answer by clear diagrams, and indicate the optical principles involved. (Lond. H.S.C.)

23. Describe the optical arrangement of a photographic camera, indicating in general terms how and why the lens differs from a simple convergent lens.

A camera has a convex lens of 7 in. focal length mounted 3 in. in front of a concave lens of 6 in. focal length. Compare the size of the image of a distant object formed by this combination with the size of the image which would be obtained if the convex lens alone was used. (N.)

24. Give an account of the optical system of a simple type of photographic camera.

What are the essential requirements of a good photographic lens? (O. & C.)

25. Explain (*a*) why telescope objectives are constructed of two or more lenses made of different kinds of glass, (*b*) why a camera lens working at the aperture $f/3{\cdot}5$ is better but more expensive than one of the same focal length working at $f/8$, (*c*) the advantages of prism binoculars as compared with those of the Galilean type. (N.)

CHAPTER VIII

THE SPECTROMETER

One of the most important instruments used in the study of light is the spectrometer. It has already been pointed out that for a transparent body the refractive index varies with the colour or wave-length of the light. By means of the spectrometer it is possible to find the exact wave-lengths of the light emitted by luminous bodies, and so to deduce their composition. Also, we can find the wave-lengths absorbed by various substances. In short, spectroscopy—which is a whole science in itself—depends entirely on the spectrometer for all measurements.

The spectrometer consists essentially of three parts—a collimator, a turntable and a telescope. The collimator is usually fixed, while the telescope can be rotated in a horizontal plane about the axis of a circular scale which is concentric with the turntable. A vernier is attached to the telescope and moves over the scale, which is divided into degrees and minutes. The collimator is a metal tube having an adjustable slit at one end and an achromatic

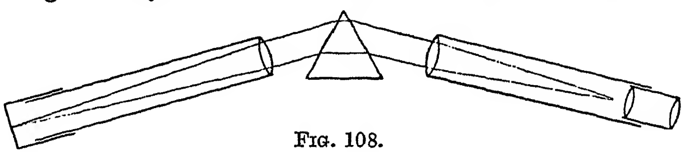


Fig. 108.

convex lens at the other. The slit is composed of two small metal plates with sharp edges, the distance between which can be varied by use of a screw. The telescope is of the simple astronomical type, with cross-wires and a Ramsden's eyepiece. The turntable may also be rotated about the axis of the circular scale, and its position can be read off by means of a vernier.

Adjustments of the Spectrometer.—1. The eyepiece is moved towards or away from the cross-wires until these are clearly visible.

2. The telescope is adjusted so that a real image of a distant object is formed at the cross-wires, *i.e.* there is no parallax between this image and the cross-wires. After this adjustment the eyepiece may be moved, as long as the cross-wires remain in the same position relative to the objective.

3. The slit is illuminated, the telescope and collimator adjusted to be in line, and the slit is then moved in or out until there is no parallax between its image and the cross-wires. Then the slit is at the focus of the collimator lens, and the light rays passing over the turntable into the telescope are parallel.

Adjustment of the Prism.—Two of the faces of a prism are usually polished, the others being left slightly roughened, and the angle between the polished faces is called the angle of the prism. Its value can be determined by either of two methods. The prism must first be adjusted so that the polished faces are vertical. This adjustment is carried out by means of three levelling screws fixed to the turntable. Let these screws be X, Y and Z, and let ABC be a principal section, A being the angle, of the prism. The prism is placed so that one of the faces bounding the angle is perpendicular to the line joining two of the levelling screws. Thus AC is shown at right angles to the line joining screws Y and Z. Generally, lines are drawn or marked on the table parallel to YZ and the above adjustment is made by turning the prism until the reflections of the lines seen in the face AC appear continuations of the lines themselves. Then the table is

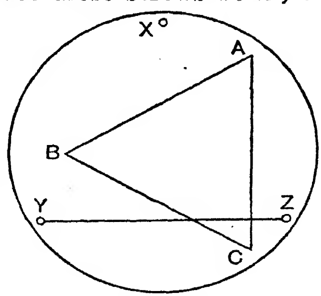


FIG. 109.

turned so that the edge A faces the collimator and light falls on the faces AB and AC. The light reflected from AC is viewed through the telescope, and any of the levelling screws adjusted until the centre of the image of the slit is just at the point of intersection of the cross-wires. The telescope is then turned to view light reflected from AB, and now only the screw X is adjusted in order to bring the image to the same position on the cross-wires as before. This adjustment merely rotates AC in its own plane, since it rotates the table about the line YZ and therefore does not change the position of the image formed by reflection from this face.

Measurement of the Angle of the Prism.—Method (1). The prism-table is turned so that light from the collimator falls on both the faces bounding the angle of the prism, and the table is then fixed. The telescope is rotated until an image of the slit, formed by reflection at one of the faces, is obtained on the cross-wires. The reading on the telescope vernier is noted and the telescope is then turned until the image formed by reflection at the other face is on the cross-wires. This reading is taken and it can be seen, as follows, that the angle turned through by the telescope is twice the angle of the prism.

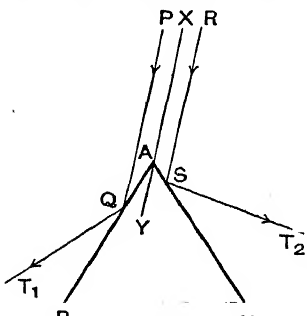


Fig. 110.

Let PQ and RS be rays incident on the faces AB and AC of the prism, and let the reflected rays be QT_1 and ST_2. Through A draw a line XY parallel to the incident rays. Then

$$T_1\hat{Q}B = P\hat{Q}A = Q\hat{A}Y \quad (\text{as PQ is } \|^l \text{ to XY})$$

and

$$T_2\hat{S}C = R\hat{S}A = S\hat{A}Y \quad (\text{as RS is } \|^l \text{ to XY}).$$

$$\therefore\ T_1\hat{Q}B + T_2\hat{S}C = Q\hat{A}Y + S\hat{A}Y$$
$$= Q\hat{A}S.$$

$\therefore$ Angle through which telescope is turned

$$= T_1\hat{Q}B + B\hat{A}C + T_2\hat{S}C$$
$$= Q\hat{A}S + B\hat{A}C$$
$$= 2A, \quad \text{where A is the angle of the prism.}$$

Method (2). The telescope is kept fixed and the prism-table is turned so that light is reflected from the face AB into the telescope and an image of the slit is formed on the cross-wires. The prism-table is then rotated until an image of the slit, formed by reflection from the face AC, appears on the cross-wires. Then the angle through which the prism has been turned is the supplement of the angle of the prism. This follows since AC is moved into a position parallel to that previously occupied by AB, and so the angle of rotation is BAD, where AD is a continuation of AC. Thus the prism is turned through an angle

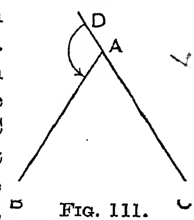


FIG. 111.

$$BAD = \pi - A.$$

Determination of Refractive Indices.—When light passes through a prism, deviation occurs and the direction of the ray is changed. The magnitude of the deviation depends on the refractive index of the prism and on the colour of the light used. In order to find the refractive index of the material of the prism, light of one colour must be used, and a sodium flame is most generally employed. This is very simply obtained by fixing a piece of asbestos, soaked in salt solution, in the edge of a bunsen flame. The slit is illuminated by means of this flame and the telescope is turned to view the light which has passed through the prism. In order to find the image it is best to have the slit wide and to look straight at the prism without the telescope. Then, when the image is obtained, the telescope may be used and the slit narrowed before any measurements are made. It is necessary to take a special

case in order to find the refractive index, for the expression to be used is only proved when the deviation of the light has its minimum value. The prism-table is therefore turned slowly to reduce the deviation, and the image is kept in view by rotating the telescope in the same direction. When a certain position is reached it is found that if the table is turned in either direction the deviation is increased. This is the position for minimum deviation and the reading of the telescope vernier is noted. The prism-table is then turned so that the deviation is produced in the opposite direction and the position for minimum deviation found as before.

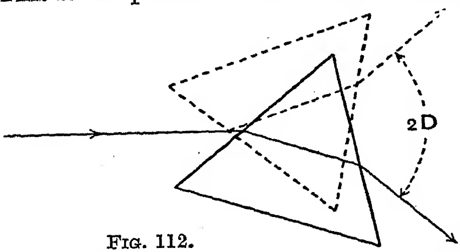


FIG. 112.

The difference between the two readings of the telescope vernier gives twice the deviation produced. If this is called 2D, then

$$\mu = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$$

so that μ, the refractive index, may be calculated.

It is possible in this manner to determine the refractive indices of liquids, the liquids being placed in a hollow prism, which has parallel-sided glass plates for the two sides bounding the angle.

Proof of Formula for Minimum Deviation.—Experiment shows that there is only one angle of incidence for which minimum deviation occurs. Since the direction of a ray of light is reversible, it follows that the angle of emergence must equal the angle of incidence for the ray which suffers minimum deviation. Thus the ray which suffers minimum

deviation passes through the prism in a direction perpendicular to the bisector of the angle of the prism.

Let PQRS be the ray, KL and LM being the normals.

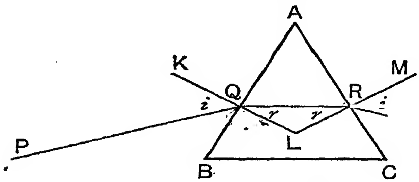


Fig. 113.

Also let the angles of incidence and emergence be i, and the angles between the normals and the ray in the prism be r. Deviation at first refraction

$$=(i-r)$$
$$=\text{Deviation at second refraction.}$$

$$\therefore\ \text{Total deviation} = D = 2(i-r).$$

Now

$$K\hat{L}M = \pi - A,$$

also

$$K\hat{L}M = \pi - 2r,$$

whence

$$A = 2r.$$

Thus

$$D = 2i - A$$

or

$$2i = A + D.$$

But

$$\mu = \frac{\sin i}{\sin r},$$

or by substitution for i and r,

$$\mu = \frac{\sin \frac{A+D}{\ }}{\sin}$$

Deviation produced by Prism of Small Angle.—When the angle of the prism is a small one, the angle of incidence for minimum deviation must be small in order that the ray may pass through symmetrically. Hence we may write

$$i = \mu r$$

or

$$\frac{A+D}{2} = \mu \frac{A}{2}$$

or

$$D = A(\mu - 1).$$

Instead of using the spectrometer, the refractive index of a prism may be found by use of pins, the same formula being employed. Two pins are used to define the incident ray, and two more pins are placed on the opposite side of the prism so that the four pins appear to be in a straight line. Then the angles of incidence and deviation may be measured, and by taking a series of readings and drawing a graph the angle of minimum deviation may be found. The angle of the prism is measured directly by means of a protractor and substitution in the formula gives a value for μ. This method, of course, is not nearly so accurate as the spectrometer one, in which the value of μ should be obtained correct to three decimal places.

The Direct Vision Spectroscope.—It has already been pointed out that when white light passes through a convex lens the image formed is slightly coloured owing to chromatic aberration, and it was shown that two lenses could be combined in order to eliminate dispersion and yet could still retain the other properties of lenses. In the direct vision spectroscope two prisms of different types of glass are arranged in such a manner that while dispersion still occurs, the deviation produced by one prism is exactly balanced by the deviation caused by the other. If the prisms are of small angle, then $D_1 = A_1(\mu_1 - 1)$ for the first prism and $D_2 = A_2(\mu_2 - 1)$ for the second. For no resultant deviation

$$D_1 + D_2 = 0$$

or

$$\frac{A_1}{A_2} = -\frac{\mu_2 - 1}{\mu_1 - 1}.$$

The minus sign before the term on the right merely indicates that the prisms must be placed so that they produce deviations in opposite directions.

In the case of prisms of larger angle the same reasoning applies, and an equation can be obtained relating the angles of the prisms to their refractive indices. Thus the two prisms A and B in the diagram can be arranged so as to give no deviation. In practice, however, the compound prism is symmetrical, often consisting of five prisms joined as shown. The shaded prisms may be of hard flint and

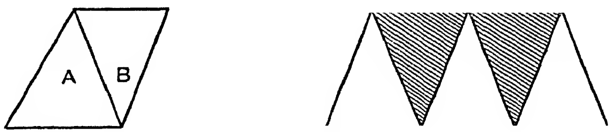


FIG. 114.

the other prisms of crown glass. So the crown glass prisms deviate the light in one direction and the flint prisms deviate it to an equal extent in the opposite direction.

Achromatic Combination of Prisms.—Sometimes it is required to join two prisms of different materials so that deviation without dispersion results. Such a combination of prisms is termed achromatic, since no colours are produced if the initial light is white. In most cases it is impossible to eliminate dispersion entirely, but it may be cut out for any two given wave-lengths. Suppose the angle of the first prism is A and the refractive indices of the material are μ_r and μ_b for red and blue light respectively. Then the respective deviations are

$$(\mu_r - 1)A \quad \text{and} \quad (\mu_b - 1)A$$

for a small-angled prism. The angle between these rays

is thus $(\mu_b - \mu_r)A$. The second prism, for which the refractive indices are μ_r' and μ_b', is arranged to deviate the light in the opposite direction, and must produce the same angular separation as before in order to eliminate dispersion. Since the angles of deviation at this prism are $(\mu_r' - 1)A'$ and $(\mu_b' - 1)A'$ respectively, where A' is the small angle of the prism, the angle between the rays is $(\mu_b' - \mu_r')A'$. So the condition for achromatism is

$$(\mu_b - \mu_r)A = (\mu_b' - \mu_r')A'.$$

The Auto-Collimating Spectrometer.—In this form of spectrometer, which was designed by Abbe, the telescope also acts as the collimator. Light enters at right angles to the axis of the telescope at a point between the eyepiece and the objectivè, but nearer to the former. This light is reflected along the telescope by a totally reflecting prism P and passes through a slit S. The prism and slit occupy

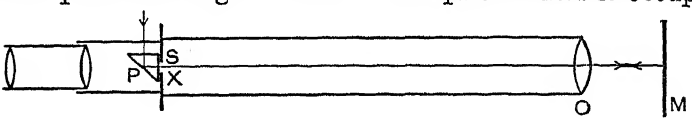


FIG. 115.

only half of the cross-sectional area of the telescope. Cross-wires or a pointer, X, are fixed immediately above the slit and the eyepiece is focussed on to these. A plane mirror, M, is placed on the prism-table beyond the objective O and the light issuing is reflected back into the telescope. The tube containing the slit, prism and pointer is movable and is adjusted until a clear image of the slit is obtained. Then S is in the focal plane of O and light strikes the mirror normally and is reflected back along the same path.

In order to measure the angle of a prism, the prism is placed on the table which is rotated so that light falls normally on one of the faces bounding the angle. The table is again rotated until light from the other face bounding the angle is reflected back along its own path.

Then the table has been turned through an angle DAE, *i.e.* the angle between the normals to these faces. Thus $B\hat{A}C = A = (\pi - \text{angle table is turned through})$.

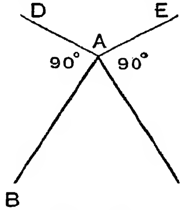


FIG. 116.

The determination of μ for the prism is then easily carried out. Monochromatic light is used to illuminate the slit, and the table is rotated so that the light is incident on the face AB, suffers refraction and meets the face AC

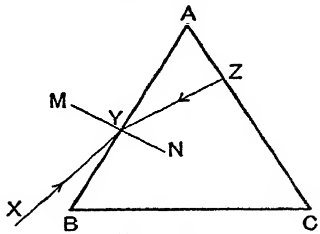


FIG. 117.

at right angles and so retraces its own path. If the ray is XYZ and MN is the normal at Y, then $N\hat{Y}Z = B\hat{A}C =$ angle of the prism (A). Now $\mu = \dfrac{\sin X\hat{Y}M}{\sin N\hat{Y}Z}$, and it is necessary to determine $X\hat{Y}M$. This may be done very simply by rotating the table until the light strikes face AB normally. The angle the table is turned through equals $X\hat{Y}M$; hence μ may be calculated.

This instrument is extremely simple to use and gives very accurate results when employed for measuring either the refractive index of a glass prism or the wave-length of light by a diffraction grating.

Determination of the Dispersive Power of a Substance in the Form of a Prism.—It has already been pointed out that the dispersive power of a substance is equal to $\frac{d\mu}{\mu - 1}$. By use of the spectrometer a very accurate value for this may be obtained. Here, either a hydrogen tube, which has prominent lines in the red and blue parts of the spectrum, or a neon lamp is used as the source of light. The reading of the telescope is taken when focussed first on to one line, and then on to the other, the deviation being a minimum in each case. The deviations are known for the two wave-lengths and by finding the angle of the prism in the manner already outlined, the two values of μ may be obtained. The average value for μ can then be found or a separate experiment may be carried out for this using sodium light (bright yellow lines) as the illuminant.

The dispersive powers of liquids can be conveniently found by this method, using hollow glass prisms to contain the liquids.

EXAMPLES ON CHAPTER VIII

1. Give a diagram showing the essential features of a simple form of spectroscope, and indicate the course of a pencil of monochromatic light through the instrument. How is it known that the visible spectrum does not represent the whole of the radiation from a high temperature source? (Lond. Inter.)

2. Being provided with a source of light, a slit, two convex lenses, each of 20 cm. focus, and a prism, describe exactly how you would form a pure spectrum upon a screen. Draw a careful diagram of the arrangement and use it to show how the spectrum is formed. (Lond. Inter.)

3. Give a short account of the construction and applications of the spectroscope. (Camb. Schol.)

4. Describe the spectrometer method of measuring the refractive index of a substance, and prove that the ray passing symmetrically through a prism undergoes minimum deviation. (Lond. H.S.C.)

5. Describe briefly (*a*) the optical arrangement of a spectroscope, illustrating your description with a diagram showing the paths through the instrument of the rays which form the ends of the visible spectrum when the slit is illuminated with white light, (*b*) how the spectroscope is used to measure the angle of a prism.

A spectroscope is set up correctly, and the readings of the telescope vernier when the cross-wires are set on the ends of the visible spectrum are observed to be 60° 45′ and 63° 15′ respectively. The focal length of the object-glass of the telescope is 27 cm., and that of the eye lens 3·0 cm. Determine the angle which the spectrum seen through the telescope subtends at the eye lens. (N.)

6. Describe carefully the readings you would make with a spectrometer in order to determine the refractive index of a glass prism.

The refractive index of a flint glass prism is 1·6434 for a red line in the lithium spectrum, and 1·6852 for a violet line in the mercury spectrum. Calculate the angle of minimum deviation for each line if the angle of the prism is 60°. (N.)

7. Describe the optical system of the spectrometer, and the adjustments you would make in setting up the instrument to measure the refractive index of a prism.

Find the relation between the angle of the prism, the minimum deviation produced, and the refractive index of the material. (O. & C.)

8. A thin prism has an angle of 10°, and its refractive index is 1·55 for red light and 1·57 for violet light. What is the angle between the emergent red and violet rays?

9. A ray of light is incident at 45° on a glass prism of angle 60°. If $\mu = 1{\cdot}5$, find the angle the emergent ray makes with the normal.

10. Show that if the angle of a prism is greater than twice the critical angle for the material of the prism then no light can pass through the prism without internal reflection taking place.

11. A ray of light is incident on a prism of angle 90°. What is the smallest value for the angle of incidence for which there will be no internal reflection if the refractive index is μ?

12. Find an expression relating the angle of a prism with the refractive index of the material, and the minimum deviation produced when a ray of light passes through the prism. The angles of minimum deviation for sodium and lithium light are 51·6° and 50·8° respectively for a prism of angle 60°. Calculate the refractive indices for the prism for these two lights.

13. A ray of light is incident normally on a glass prism of angle 36°. What deviation is produced if $\mu = 1{\cdot}5$?

If the prism is now placed in water ($\mu = 1{\cdot}33$), what will be the deviation produced for normal incidence?

14. What do you understand by the term "dispersion of light"?

Explain how it is possible to construct a prism either for the purpose of (*a*) deviating light without dispersing it, or (*b*) dispersing light without altering the direction of the central ray? (O. & C.)

15. Define the deviation and the dispersion of light as produced by a glass prism. Explain the principle of the direct-vision spectroscope. (Lond. Inter.)

16. Describe a good method of measuring the refractive index of a substance such as glass, and give the theory of the method.

A glass prism of angle 72°, and index of refraction $1{\cdot}66$, is immersed in a liquid of refractive index $1{\cdot}33$. What is the angle of minimum deviation for a parallel beam of light passing through the prism? (Lond. H.S.C.)

17. How would you measure the angle of minimum deviation of a prism?

(*a*) Show that the ray of light which enters the first face of a prism at grazing incidence is least likely to suffer total internal reflection at the second face.

(*b*) Find the least value of the refracting angle of a prism made of glass of refractive index $7/4$, such that no rays incident on one of the faces containing this angle can emerge from the other face. (N.)

18. Describe carefully how you would determine the refractive index of a glass prism.

A glass prism with a refracting angle of 60° has a refractive index of $1{\cdot}515$ for red light and $1{\cdot}532$ for violet light. A parallel beam of white light is incident on one face at an angle of incidence which gives minimum deviation for red light. Determine (*a*) this angle of incidence, (*b*) the angular width of the spectrum, (*c*) the length of the spectrum if it is focussed on a screen by an achromatic lens of 100 cm. focal length. (N.)

19. Explain carefully how the deviation of a parallel beam of light by a glass prism varies with the angle of incidence of the beam on the first surface of the prism. How does it depend for a particular angle of incidence on the angle of the prism, on the refractive index of the glass and on the colour of the light? (Lond. Inter.)

20. Define dispersive power.

The following table gives the refractive indices of crown and flint glass for three lines of the spectrum.

	C.	D.	F.
Crown	1·514	1·517	1·523
Flint	1·644	1·650	1·664

Calculate the refracting angle of a flint glass prism which, when combined with a crown glass prism of refracting angle 5°, produces a combination that does not deviate the light corresponding to the D line. What separation of the rays corresponding to the C and F lines will such a compound prism produce? (Lond. H.S.C.)

21. Prove that, for a prism of small angle A, the deviation of a ray of light is $(\mu - 1)$ A, provided the angle of incidence also is small.

A crown glass prism, with a refracting angle 6°, is to be achromatised for red and blue light with a flint glass prism. Using the data given below, and assuming that the formula given in the first part of the question applies, find (*a*) the angle of the flint glass prism, (*b*) the mean deviation.

		Crown Glass	Flint Glass
Refractive Index (μ)	Red	1·513	1·645
	Blue	1·523	1·665

(N.)

CHAPTER IX

SPECTRUM ANALYSIS

ALTHOUGH it had been known for many centuries that colours could be produced from white light in certain circumstances, it is to Newton that we owe the first investigation of spectra. The term "spectrum" was given by Newton to the band of colours formed when white light was passed through a prism. The historic experiment was carried out in 1666, at Cambridge, when light rays from the sun passed through a circular hole in a shutter, and after being refracted through a prism, formed a spectrum on a screen. The colours of the spectrum gradually merged from one to the other, there being no distinct boundaries between them. In order from the red end the colours are red, orange, yellow, green, blue, indigo and violet, although many observers fail to distinguish indigo as a separate colour.

Further experiments were carried out by Newton to discover whether or not the light could be further split up. The spectrum was formed on a screen in which was a narrow slit so that some rays of a definite colour might pass through to a second prism. It was found that after passing through this second prism on to a screen, the colour

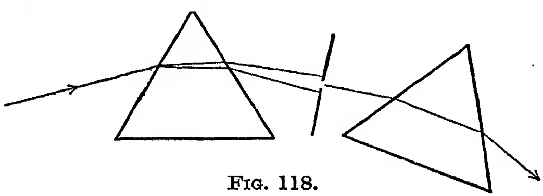


FIG. 118.

was unchanged and no further colours were introduced. Another experiment of a similar nature was carried out, the second prism in this case being placed with its refracting edge at right angles to that of the first prism. The

prisms are then said to be "crossed." The resulting image was a very broad spectrum—each colour was spread out in a direction at right angles to the length of the spectrum, but no further decomposition of the light took place.

This investigation led Newton to the conclusion that white light could be split up into seven colours, and that each of these colours was fundamental and could not be split up any further.

Recombination of the Spectral Colours.—If a spectrum is formed in the usual manner and the screen is replaced by a prism of the same angle as that forming the spectrum, but

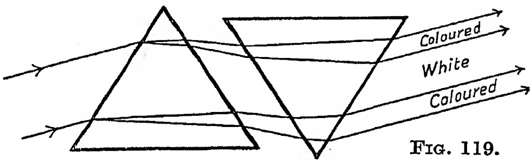


FIG. 119.

deviating the light in the opposite direction, the adjacent coloured beams overlap and produce a beam of white light except at the edges where complete overlapping does not occur and colours still persist. Newton showed this effect of recombining the colours by using a lens instead of the second prism mentioned above. Another method of synthesising white light is to throw the spectrum on to seven small plane mirrors arranged in an arc. Each

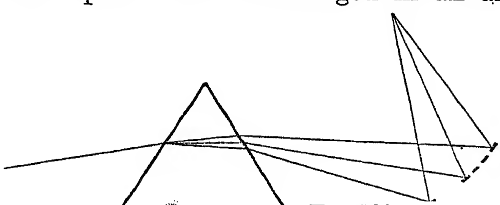


FIG. 120.

colour is reflected from its mirror and a screen placed in the focal plane of the mirrors shows a patch of white light.

Formation of a Pure Spectrum.—At this point it may be instructive to interrupt the historical order of spectral

analysis in order to consider how a clear spectrum is obtained. The essentials are (1) a narrow slit (illuminated by the source of light) parallel to the edge of the prism; (2) a lens in order to give a parallel beam; (3) a prism in the position of minimum deviation; (4) a convex lens to

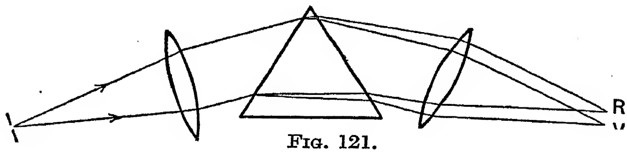


FIG. 121.

form the image on a screen, or a telescope if the light is to be viewed directly. For each colour all the rays are parallel and thus the second lens brings all the rays of one colour to the same point.

In order to obtain greater dispersion and a longer spectrum, two prisms—not in contact—are used and each is set in the position for minimum deviation for yellow light.

It will be noticed that Newton did not use a slit but preferred a circular opening. This meant that the spectrum so formed was not a pure one, since owing to the size of the opening a number of spectra was formed, each of them clear, but all overlapping. The introduction of the slit is attributed to Wollaston, although Newton used it in some experiments, while the first continuous use of lenses in the formation of the spectrum was by Fraunhofer. Both these advances were made in the early part of the nineteenth century.

The Fraunhofer Lines in the Solar Spectrum.—Fraunhofer is chiefly notable here for the work he did in connection with certain dark lines which can be seen in the spectrum of the sun. Wollaston had already noticed these lines some years earlier (1802), but Fraunhofer (1814) was the first to show that they were fixed in position—always occupying the same places in the solar spectrum. To make quite sure that the lines were due to the sun, Fraunhofer

experimented with different prisms of different materials, some glass and some liquid, and he found that the lines always occurred in the same positions and differed in intensity. He lettered the chief lines and noted their positions, and also observed that the same lines appeared in the spectra of the moon and the planets. In the case of stars, dark lines were found in their spectra, but not in the same places and not of the same intensity. This fact showed that the lines in the solar spectrum could not be all produced by the earth's atmosphere.

Now, although Fraunhofer introduced the familiar sodium flame as a source of monochromatic light, and also observed that there was a bright line between the orange and the yellow in the spectrum of a flame in exactly the same position as one of the lines in the solar spectrum, the D line, he was unable to explain the cause of the solar lines. It was not until 1849 that Foucault solved the problem, but, as little notice was taken of this investigation, the explanation of the solar lines is often attributed to Kirchoff who, in 1859, rediscovered the principle already put forward ten years previously. Foucault formed an image of the sun on the arc between charcoal poles, and examined the light by a spectrometer. The arc spectrum consisted of a number of luminous lines amongst which a double line between the yellow and the orange stood out strongly. When the two spectra were superimposed exactly, the dark D line in the sun's spectrum was strengthened, while if the spectra were arranged one above the other, the D line in the solar light still appeared and coincided exactly with the bright line in the light from the arc. Thus Foucault concluded that the arc emitted rays of this wave-length itself and at the same time absorbed them when they came from another source.

The same effect is shown easily by using a lantern as a source of white light, and shining the light direct on to the slit of a spectrometer. A sodium light—formed by holding a small piece of asbestos soaked in salt solution

in the edge of a bunsen flame—is placed in the path of the light and is focussed on to the slit. The resulting spectrum shows the ordinary continuous spectrum crossed by a double dark line in the orange-yellow portion. If the lantern is extinguished the dark line appears bright by reason of the sodium light. The effect is known as the reversal of the D line.

Kirchoff, in his investigation, carried the matter a step further than Foucault, and in conjunction with Bunsen stated that when a substance emits light, the light is of definite wave-lengths characteristic of that substance, and also the substance is capable of absorbing light of exactly the same wave-lengths. Thus the dark lines in the spectrum of the sun are caused by the vapours surrounding the sun absorbing the light of particular wave-lengths. Kirchoff also found that the dark line is only dark by comparison with the light on each side of it, and really is bright to some extent. This is an example of resonance, which is the gain of energy of one body from another if the periods of vibration of the bodies are the same. It may be illustrated by suspending two simple pendulums of equal length from a cord stretched horizontally. When one pendulum is set in vibration, the other, which is initially at rest, takes up the motion and vibrates in the same period as the first one, showing that energy is transferred from one pendulum to the other. In the optical case energy is taken from the waves of a certain period in the white light and absorbed by the sodium, which could give rise to vibrations of exactly the same period if it were excited itself. So we may state that *the radiation emitted by a body when incandescent is absorbed by the same body when cold*, where the word cold is used in the comparative sense.

The composition of the sun is thus found. The central part is liquid or solid at a very high temperature, and gives out white light. This is surrounded by layers of cooler vapours which absorb light of certain wave-lengths

which they would emit if incandescent. The hot nucleus is termed the photosphere, and the outer layers of vapours form the chromosphere. Knowing the wave-lengths missing from the solar spectrum and comparing these with the wave-lengths emitted by various elements, it has been found that the sun's atmosphere consists of vapours of many of the elements existing on the earth. A table of the chief Fraunhofer lines is given below, and shows the elements whose spectra correspond with the different lines. The wave-lengths are given in Ångström units, which are often referred to as angstroms or A.U.

The whole spectrum was mapped by Ångström, and the wave-lengths of a large number of the Fraunhofer lines found and expressed in 10^{-8} cm. Thus 1 Ångström unit equals 10^{-8} cm.

TABLE OF FRAUNHOFER LINES

Line	Substance	Wave-length	Line	Substance	Wave-length
A	*	7661	*h*	H (δ)	4102
B	*	6867	H	Ca	3968
C	H (α)	6563	K	Ca	3934
D	Na	5896, 5890	..	Mg—C	3838
E	Fe	5270	L	Fe—C	3820
b	Mg	5178, 5173	M	Fe	3720
F	H (β)	4861	N	Fe	3581
f	H (γ)	4340	O	Fe	3441
G	Fe	4308	..	Ti—Fe	3057

* Due to absorption by the oxygen in the earth's atmosphere.

It should be pointed out that some of the Fraunhofer lines are due to absorption by the earth's atmosphere. More lines are observed if the sun is near the horizon than if it is well up in the sky. These lines are due chiefly to oxygen and water-vapour.

The spectra of the stars may be examined in a similar manner to that applied in the case of the sun, but here, owing to the appearance of the stars as points, a collimator is not necessary. In general the spectra fall into two groups—those resembling the solar spectrum with its dark lines, and others whose spectra are fluted.

The work of Kirchoff and Bunsen is important, not only in connection with the solar spectrum and the composition of the sun, but also in giving us a method of detecting and discovering elements. The spectra of the known elements are mapped, and if another substance gives lines not contained in any of the other spectra, this substance must contain a new element. Cæsium, indium, rubidium and thallium were each discovered in this manner, while helium, which gives a line, the D_3 line, very close to the sodium lines, was discovered to exist on the sun some years before it was isolated on the earth.

Types of Spectra.—Spectra may be divided into two types: emission spectra and absorption spectra, each of which may be subdivided. The first type may be produced by

(*a*) heating the substance in a bunsen flame;
(*b*) passing an electric discharge through the rarefied gas in a discharge tube;
(*c*) placing a small quantity of the substance in the crater of the positive electrode of an arc and passing a current;
(*d*) passing a discharge between metal electrodes, the spectrum in this case depending on the metal and also on the air or gas surrounding the electrodes.

Emission Spectra

Continuous Spectrum.—This spectrum is emitted by any solid body when white-hot, and it contains all the colours merging from one to the other in the order observed by Newton. An ordinary luminous gas flame gives a con-

tinuous spectrum due to the incandescent particles of carbon in it. This continuous spectrum is caused by the molecules in the solid being very close to each other, so that collisions are constantly occurring and all the possible vibrations and wave-lengths are emitted.

Line Spectrum.—Elements, when heated to incandescence if solid, or when exposed to an electric discharge if gaseous, form this spectrum which consists of sharp bright lines. The arrangement of lines may be regular, in which case it is termed a "series," *e.g.* Balmer's series for hydrogen, or the lines may be spaced quite irregularly.

Band Spectrum.—This consists of a number of bright bands of light, one edge of the band being sharp, the other edge being more difficult to determine as the intensity gradually falls off as this edge is approached. The spectrum thus has a fluted appearance and is often referred to as a fluted spectrum. If examined with a spectrometer of high resolving power, each band is seen to consist of a large number of bright lines which are very close together on the side where the edge is sharp, but get further and further apart towards the other edge (fig. 122).

This type of spectrum is emitted by compounds, unless, of course, the substance is heated too strongly and breaks up into the constituent elements. Most of the carbon compounds can be shown to give spectra of this type, and the blue part of a candle or bunsen flame shows this band spectrum. This spectrum of the bunsen flame was first noted by Swan, and is sometimes called the Swan spectrum.

FIG. 122.

It has been found that a substance can emit both a line and a band spectrum, although not simultaneously. This was first noted in the case of nitrogen, and it is probable that the line spectrum results when the gas is in the atomic state, while for the production of the band spectrum the gas must be in the molecular state. Generally, if the discharge is violent the line spectrum is obtained.

Absorption Spectra

Continuous or General Absorption.—The usual continuous spectrum is crossed by one or more dark bands which generally do not possess sharp edges. This is caused by interposing some substance, such as red glass, cobalt glass, or didymium glass, between the slit and the source of light. The red glass absorbs practically all the colours except red and orange, while cobalt glass absorbs three different groups of waves, so that the spectrum is crossed by three dark bands. Many solutions show absorption bands, and in investigating these spectra the liquid is usually contained in a glass cell with parallel sides as shown.

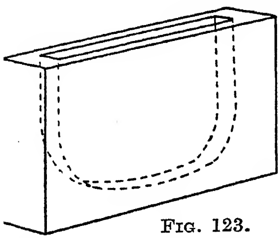


Fig. 123.

Potassium permanganate, potassium dichromate, and many of the aniline dyes are suitable liquids for examination in this manner.

Selective or Line Absorption.—In this case the spectrum is crossed by dark lines as in the case of the solar and stellar spectra. The reason for these dark Fraunhofer lines has already been explained as due to the absorption of certain wave-lengths by vapours through which the light passes.

The Doppler Effect

If the source of light which is giving out certain definite wave-lengths is in motion towards or away from the spectrometer, it is observed that the wave-lengths received are slightly different from those obtained when the source is at rest. The velocity of the source must be very great otherwise no change in the wave-length can be detected. In the case of stars, which have enormous velocities through space, this change can be observed. This effect is known as the Doppler effect, since the change in

wave-length due to relative motion of the source and observer was first considered by Doppler. The same effect occurs in sound, where examples are much more frequent owing to the much smaller velocity of sound. The pitch of the whistle of an engine moving at high speed past an observer apparently drops very rapidly just as the engine passes.

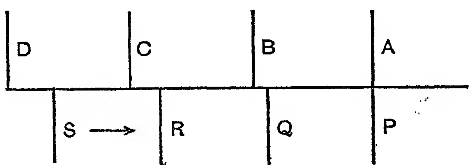
If the source moves towards the observer, who is at rest, with velocity v, and emits light of wave-length λ and frequency n, then between the emission of each wave the source moves a distance $\frac{v}{n}$, since the time between the vT emission of each wave is $1/n$. Consequently the apparent wave-length (λ'), as received by the observer, is $\left(\lambda - \frac{v}{n}\right)$,

i.e. $\lambda' = \lambda - \frac{v}{n}$.

But $n\lambda = V$, the velocity of light. $\lambda' = \lambda\left(1 - \frac{v}{n\lambda}\right)$

$$\lambda' = \lambda \frac{V - v}{}$$

Thus the wave-length is reduced if the source moves towards the observer, and a spectral line will be displaced towards the violet end of the spectrum.



Above Source at rest $DC = CB = BA = \lambda$.

Below Source in motion $SR = RQ = QP = \lambda' = \lambda \cdot \frac{V - v}{V}$.

FIG. 124.

If the source moves away from the observer then v is negative, the wave-length is increased, and the line dis-placed towards the red end of the spectrum.

When the observer is in motion, say with velocity u towards the source which is at rest, the treatment is similar. Instead of receiving n waves in one second, as he would do if at rest, the observer, by moving towards the source, receives or meets $\left(n+\frac{un}{V}\right)$ waves since he moves a distance u and there are n waves in a distance V. So the frequency of the light received is $n\frac{V+u}{V}$ and wave-length is $\lambda\frac{V}{V+u}$.

Using the same symbols, then if both source and observer are moving towards each other the recorded wave-length would be

$$\lambda\frac{V-v}{V+u}.$$

This theory was applied by Huggins in 1868 to the spectrum of Sirius in which one of the hydrogen lines was displaced towards the red, and he showed that Sirius must be moving away from the earth at a relative speed of about 30 miles per second.

If the source and observer are moving in directions at angles to the line of sight the component velocities along this line must be taken. In the case of stars emitting continuous spectra there is no change in colour because of their motions, for, as will be shown later, the spectrum continues at each end beyond the red and the violet. If the star is approaching, all wave-lengths are reduced, so that some disappear into the invisible part at the violet end, while radiations previously invisible at the red end now become visible in the spectrum.

Applications of the Doppler Effect

Double Stars.—Many stars which, when viewed through the telescope, appear to be single stars, are in reality double stars or spectroscopic binaries. These stars rotate

about each other so that as one is approaching the earth the other is receding. If the stars have approximately equal brightness all the lines observed in their spectra sometimes appear as single lines and sometimes as double lines. The first case occurs when one star is hidden by the other, and neither has any velocity towards the earth. When both stars are visible the velocity of one towards the earth causes the spectral lines to be displaced towards the violet, while the velocity of the other in the opposite direction results in a displacement towards the red end of the spectrum. So we have the second case where all the lines are doubled. This method of examination shows that a large number of the stars are double.

Saturn's Rings.—Around the planet Saturn are three flat concentric rings, and it was unknown whether the rings were solid or were made up of a large number of small individual particles. Now, if the rings were solid, we should have

$$v \propto r \quad . \quad . \quad . \quad . \quad (1)$$

where v is the linear velocity of a point distant r from the centre of Saturn, while if the rings were made up of individual particles, these particles must be in equilibrium under the influence of the centrifugal force and the gravitational force, so that

$$\frac{v^2}{r} \propto \frac{1}{r^2}$$

or

$$v \propto \sqrt{\frac{1}{r}} \qquad (2)$$

Investigation with the spectrometer showed that the Fraunhofer lines, which appear since the planet reflects the sunlight falling on it, were more displaced when the light came from the inner edge than when it came from the outer edge. This meant that the inner edge moved more rapidly than the outer edge, a result in agreement with (2).

Further investigation gave results which agreed almost exactly with this equation, thus bearing out the theory that the rings are composed of individual particles.

Measurement of the Angular Velocities of the Sun and Planets.—By receiving the light from only a small part of the surface of a planet at a time, the positions of the Fraunhofer lines may be noted and compared with the positions found when the light comes from another part of the surface. Any disparity between the results shows that all parts of the planet have not the same velocity towards the earth, and by measuring the displacements for light from different parts of the surface the angular velocity of the planet may be found. The same method can be applied in the case of the sun.

The Broadening of Spectral Lines.—Owing to the high velocities of the particles moving about in a gas, slight variations occur in the wave-lengths emitted when a gas is incandescent. Consequently spectral lines are not exactly fine, and since an increase of temperature increases the velocities of the particles, the spectral lines are broader the higher the temperature.

Atomic Structure and Emmission Spectra

According to Bohr's theory of the structure of atoms, the atom consists of a positively charged nucleus with electrons revolving around it. Practically the whole mass is in the nucleus which is comprised of a number of protons—hydrogen nuclei each bearing a positive charge —and electrons. The number of protons is the same as the atomic weight, and the number of electrons outside the nucleus is the same as the atomic number. Since a proton bears a unit positive charge and an electron a unit negative charge, then in order for the atom to have no resultant charge the number of electrons in the nucleus is numerically equal to the atomic weight less the atomic number. Taking a more modern view since the discovery

of the neutron, which has the same mass as the proton but possesses no charge, the nucleus may be considered to consist of equal or nearly equal numbers of protons and neutrons. The neutron can be taken as a proton joined to an electron, the charges having neutralised each other.

The hydrogen atom presents much the simplest case, for here we have one proton round which a single electron revolves. This electron can only move in certain orbits, the radii of which are as the squares of the natural numbers —1^2, 2^2, 3^2 . . . If the electron is pulled away from the inner orbit to an outer one its energy is increased, and now the electron may fall back to an orbit of smaller radius with the emission of energy. This energy is radiated in wave form and may cause a spectral line of a definite wave-length in the visible spectrum.

By the quantum theory, energy may only be given out in "lumps," and if the frequency of the light emitted is n, then the energy given out is hn, where h is Planck's constant. Every time the electron falls back to an orbit of smaller radius, a spectral line is produced. Bohr showed by application of the quantum theory that

$$n = \mathrm{R}\left(\frac{1}{p^2} - \frac{1}{q^2}\right),$$

where R is a constant called Rydberg's constant and p and q are whole numbers. Now in 1885 Balmer had shown that the frequencies of the lines in the spectrum of hydrogen were satisfied by the equation

$$n = \mathrm{K}\left(\frac{1}{2^2} - \frac{1}{q^2}\right),$$

where K is constant and q is a whole number greater than 2. Bohr's theory therefore agrees with Balmer's results if $p = 2$, which means that the electron is falling from some

orbit into the second orbit from the nucleus. Now if $p=1$ another series of spectral lines should result, and these lines have been observed by Lyman, not in the visible spectrum but beyond the violet. Further, a series of lines for which $p=3$ and q is a whole number greater than 3 has been discovered by Paschen in the infra-red, while certain frequencies of hydrogen lines in the infra-red agree with the formula if $p=4$.

Thus, experiment gives results in close agreement with those obtained by Bohr's theory for the hydrogen atom.

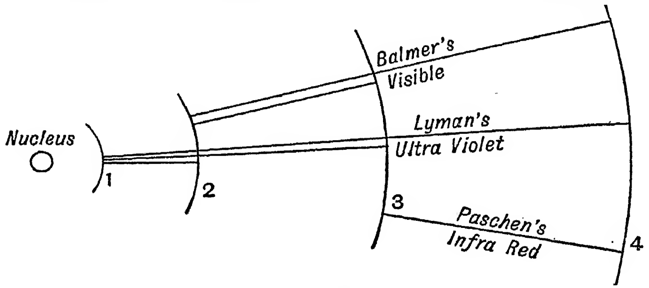


FIG. 125.

Naturally, other atoms are more complex owing to the number of electrons in orbits round the nucleus and, since the electrons exert forces on each other, the simple theory cannot be applied. But the principle that there are energy levels or orbits still holds good, and all spectral lines are caused by the emission of a quantity of energy by electrons in jumping from one level to another.

EXAMPLES ON CHAPTER IX

1. What are the distinctions between (*a*) a line spectrum, (*b*) a continuous spectrum, and (*c*) an absorption spectrum? Describe how each may be produced. (Lond. Inter.)

2. What is meant by a spectrum? How are spectra produced? State what you know of different types of spectra. (Ox. Schol.)

3. How would you obtain (*a*) a continuous spectrum, (*b*) a line spectrum, (*c*) a band spectrum, (*d*) an absorption spectrum? Describe how these spectra are formed.

4. Describe a spectrometer and the mode of adjusting it to exhibit the spectrum of the light from an electric arc. What is the characteristic difference between the spectra of sunlight and light from the arc? (Lond. H.S.C.)

5. Explain the production of dark line spectra.

Describe a method by which you could show the reversal of the sodium lines in the spectrum of an arc lamp.

6. How would you arrange an experiment to project a pure spectrum on a screen? Describe the spectrum produced by (*a*) sunlight, (*b*) an electric filament lamp, (*c*) a sodium flame. (O. & C.)

7. Explain Doppler's Principle, and discuss its application to light waves. (Camb. Schol.)

8. Mention some of the more important applications of the Doppler Effect.

It is found that the G line in the spectrum of a star is displaced, and the apparent wave-length is $4{\cdot}3074 \times 10^{-5}$ cm. If the true wave-length for the G line is $4{\cdot}3079 \times 10^{-5}$ cm., find the relative velocity between the star and the earth, if the velocity of light is 3×10^{10} cm./sec.

CHAPTER X

THE INVISIBLE RADIATIONS

So far we have considered in our study of the spectrum only the radiations, which, when received by the eye, cause the sensation of light. Actually there are far more radiations which are not received because their wave-lengths are too small or too large to affect the eye. A hot body, such as a white-hot poker, will yield not a continuous spectrum alone, but waves beyond the violet—called the ultra-violet waves, and also waves of length greater than those at the red end of the spectrum. The latter are called infra-red waves.

The Ultra-violet Spectrum.—It has been found that ordinary glass absorbs most of the ultra-violet waves, and consequently quartz prisms and lenses must be used in experiments on ultra-violet radiation. Quartz is transparent to ultra-violet light, as is Vita glass—the modern glass for windows. Fluorite and Iceland Spar can each be used, but the former is not as good as quartz optically, while the latter has two refractive indices which would result in two spectra being obtained. Quartz also is a doubly refracting substance, but to a much smaller degree than Iceland Spar. This double refraction may be overcome by the use of a Cornu prism in which two similar prisms are joined so that they act in conjunction. But one prism is of right-handed quartz and the other of left-handed quartz, so that the double refractions balance and a sharp spectrum results.

The chief methods used for the *detection* of ultra-violet rays are:

(1) Photography, (3) Phosphorescence,
(2) Fluorescence, (4) Photo-electricity.

(1) If the light from an arc lamp passes through a quartz prism and forms a continuous spectrum on an ordinary photographic plate or on a piece of gaslight printing paper, the plate or paper is blackened (after development) in the blue and violet parts of the visible spectrum and also in the part beyond the end of the violet. It was by the blackening of silver chloride that the existence of ultra-violet waves was discovered at the commencement of the nineteenth century. If the sun is the source of light the ultra-violet spectrum is again produced and is crossed by Fraunhofer lines.

(2) A white filter paper soaked in quinine sulphate solution, slightly acidulated with sulphuric acid, and then dried, may be substituted for the photographic plate, and in this case the paper beyond the violet gives out a blue light—practically a forget-me-not shade of blue. This phenomenon is called *fluorescence*. The ultra-violet radiations are absorbed by the quinine sulphate and light of a longer wave-length is radiated. In most cases of fluorescence the ultra-violet and the violet rays are those which suffer this change in wave-length. Certain other substances also fluoresce, and Stokes, who was responsible for most of the initial work on this subject, was led by his investigations to the result that the wave-length of the radiation absorbed is always less than that of the fluorescent radiation. This means that the frequency of the light is reduced and in accordance with the quantum theory, the loss in energy, or the energy absorbed by the fluorescent substance, is $h(n_a - n_e)$ where h is Planck's constant and n_a and n_e are the frequencies of the light absorbed and emitted. The fall in the frequency has led to the phrase "degradation of light" in this connection. More recent investigation has revealed that Stokes's law is only a generalisation and that in some cases the waves emitted have a greater frequency than those absorbed. Generally in these cases some chemical action takes place. A list of common fluorescent substances is given overleaf. A piece

of uranium glass, which fluoresces green, is sometimes placed in the telescope of an ultra-violet spectrometer so that the ultra-violet rays may be analysed directly.

Substance	Actual colour	Fluorescent light
Quinine sulphate	Colourless	Blue
Fluor spar	Colourless	Violet
Fluorescein	Brown	Green
Vaseline	Yellow	Green
Barium platinocyanide	Yellow	Green
Uranium oxide	Yellow	Green
Chlorophyll	Green	Red

This phenomenon of change in colour was called fluorescence since the effect was first noticed in the case of fluor-spar.

(3) In fluorescence the light emitted ceases immediately the exciting light is cut off, but in another similar phenomenon the substances continue to emit light after the initial light or radiation is withdrawn. The name given to this effect is *phosphorescence*. Experiments show that phosphorescence is caused chiefly by the ultra-violet and violet parts of the spectrum, and since fluorescence usually accompanies phosphorescence, this may be used as a method of detecting and investigating the ultra-violet radiations. The commonest phosphorescent substances are calcium sulphide, barium sulphide and strontium sulphide. If a screen is coated with calcium sulphide and a continuous spectrum is formed on it, then on cutting off the spectrum the screen still emits light of a pale blue shade from the parts previously illuminated by the violet and ultra-violet radiations. Other colours may be obtained by using suitable mixtures of phosphorescent substances. Phosphorescence received its name from the action of phosphorus in emitting light when placed in a dark room. Actually this light is due to chemical action and has no connection with true phosphorescence.

Becquerel carried out investigations on a large number of substances and he found that most of them were phosphorescent although the degree of phosphorescence varied greatly. Whereas a substance such as calcium sulphide will glow for nearly an hour after being illuminated, other substances will only phosphoresce for a very small fraction of a second. In his phosphoroscope, Becquerel arranged that sunlight should be converged to a focus by a lens L and the substance to be examined was placed in the focal plane of this lens. Between the lens and substance was a disc A containing a number of circular holes, and on the other side of the substance was a similar disc B mounted on the same axle as A. These discs were arranged so that they were exactly out of step, *i.e.* when light passed through A to the substance, no light could pass on through B. Then as the discs were rotated the substance was illuminated by light passing through a hole in A, and later, when no light passed through A, the substance could be seen through an opening in B. So the substance was not seen when illuminated, but at some time later, and by varying the speed of rotation of the discs the duration of the phosphorescence was found.

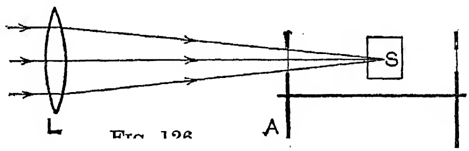


Fig. 128

(4) If a metal plate is exposed to light it is found to acquire a small positive charge. The effect is called the *photo-electric effect* and is due to the emission of slow-moving electrons from the plate. If a sheet of glass is placed between the plate and the source of light, the effect is stopped in most cases, thus showing that it is due to the ultra-violet radiations which are now absorbed by the glass plate. So if a plate is joined to a quadrant electrometer or sensitive electroscope a deflection will be caused when ultra-violet rays fall on the plate.

We have thus four methods available for detecting and

investigating ultra-violet radiations, and it becomes a simple matter to determine the degree to which substances transmit or absorb these invisible rays. Flint glass is opaque to the rays, ordinary window glass and cobalt blue glass are partly transparent, while quartz is quite transparent. If a mirror is held in the path of ultra-violet light, it reflects this light and the reflected rays can be traced out by use of a fluorescent substance. Refraction can be shown by passing the rays through a quartz prism which deviates the rays to another part of the screen. Polarisation can be observed by passing the rays through two Nicol prisms on to a fluorescent screen which will appear bright and dark in turn as one of the nicols is rotated. Ultra-violet light also interferes in exactly the same manner as visible light. This was proved conclusively in 1811 by Young who obtained Newton's Rings with ultra-violet rays, the effect being shown by aid of silver chloride.

The Infra-red Spectrum.—In order to investigate the portion of the spectrum beyond the red, it is necessary to use rock salt prisms and lenses since other types of glass absorb, to some extent, the infra-red radiations. Frequently lenses are dispensed with and concave mirrors are used instead. The spectrum obtained is smaller than if quartz is used, since rock salt has a smaller dispersive power. The methods used for detecting and investigating infra-red radiations are:

(1) Heating effect,
(2) Photography, using special plates,
(3) Photo-electricity.

Of these methods the first is the most important. In (2) it has been found that special panchromatic photographic plates which contain certain isocyanides are affected by rays beyond the red, and these plates may be used in this case just as the ordinary plate is used for detecting ultra-violet radiation.

The photo-electric effect has already been considered and may be applied here if a photo-electric cell of some substance such as rubidium is used.

The detection by heating effect is of historical importance since the existence of the infra-red spectrum was first found in 1800 by Herschel, who noticed that a sensitive thermometer with a blackened bulb indicated a rise in temperature when placed beyond the red end of the spectrum. The instruments in general use now for detecting this radiation are the thermopile, the bolometer, the radio-micrometer and the radiometer.

The thermopile is based on thermo-electric action. If two wires of different metals are joined at both ends and a galvanometer is inserted in series, then any difference in the temperatures of the junctions of the wires causes a current to flow through the galvanometer. This arrangement is called a thermo-couple, and if a number of thermo-couples is arranged in series, the sensitivity is increased and we form an instrument called a thermopile. This instrument has two faces, the odd junctions being on one face and the even junctions on the other. One face is usually enclosed so that its temperature is constant while the other face is exposed to the radiation.

The bolometer, invented by Langley who carried out a good deal of experimental work on the infra-red, works on the principle that a rise in temperature causes an increase in the resistance of metals. Two very fine strips of platinum foil about $\frac{1}{2000}$th mm. in thickness form two arms of a Wheatstone bridge. One strip is enclosed and remains at a constant temperature, while the other is blackened and exposed to the radiation. Any heat rays warm up the latter strip, and the slightest heating effect upsets the balance of the bridge. This instrument is extremely sensitive, and will detect changes in temperature of one hundredth of a degree Centigrade.

Boys's radio-micrometer consists of a small coil of wire, the ends of which run below the coil to short lengths of antimony and bismuth respectively. The coil is suspended by a quartz fibre, which has a very small torsion constant, between the poles of a circular magnet as in the case of

moving coil galvanometer. The free ends of the antimony and bismuth are joined, and a blackened copper disc placed over this junction. If heat radiation falls on the disc, the heat is easily conducted through the copper to the junction of antimony and bismuth, and a thermo-electric current is set up. This causes the coil to rotate, and the deflection is noted by observing the deflection of a spot of light reflected from a mirror attached to the quartz fibre where it joins the coil.

The radiometer, invented by Crookes, is composed of movable mica vanes, one side of each vane being blackened and the other silvered. The vanes are fixed to a spindle which is lightly poised so that it can rotate easily, and the whole is contained in a glass bulb which is very nearly exhausted. When radiant heat falls on the radiometer the vanes rotate so that the black surface goes away from the source of the radiation. This occurs because the blackened surface attains a temperature slightly above that of the silvered one, so that the particles of gas in the bulb experience a slight rise in temperature on hitting the blackened surface. They therefore leave it with a slightly increased velocity, and consequently the surface recoils and the vanes rotate.

With any of these instruments the infra-red may be investigated. Results show that there is no heating effect in the ultra-violet nor in the visible spectrum until orange is reached. The effect is increased when the red is reached, but the maximum heating occurs beyond the red. If the instrument used is placed in the infra-red, and sheets of glass of different types are placed in turn between the source and the instrument, the absorptive powers of these sheets for infra-red rays may be found. Rock salt absorbs least, and then come quartz, ordinary window glass, flint glass and finally calc-spar, which is almost opaque to these long waves. A sheet of ebonite, although opaque to visible light, is transparent to infra-red rays. The infra-red spectrum is continuous like the ultra-violet and

visible spectra, and contains a number of "dark lines"—wave-lengths for which no heating effect is obtained. Infra-red rays may be reflected, refracted and polarised just as ordinary light. They travel with the same velocity as visible rays, for at solar eclipses the heat of the sun ceases to reach us as soon as the eclipse becomes total and the light fails. Consequently the solar radiations consist of waves of widely differing length. The visible spectrum has a range of nearly one octave—from 3950 to 7600 angstroms approximately—and is bounded by the ultra-violet with a range of five octaves on one side, and the infra-red with a range of about nine octaves on the other.

The curves of luminosity and radiant energy against wave-length show that the green-yellow part of the spectrum has the greatest effect on the eye, while the heat energy reaches its maximum in the infra-red for the electric lamp or the arc lamp, but in the visible spectrum

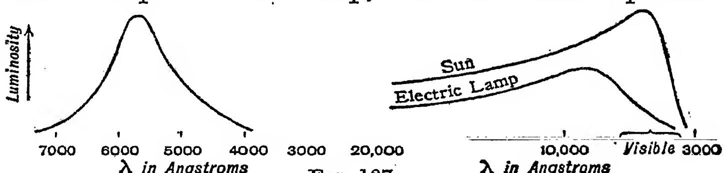


FIG. 127.

if the sun is the source. This appears to contradict the results obtained by experiment, where the thermopile or bolometer records maximum deflections in the infra-red for solar radiation. The reason for this is that the spectrum is spread out far more towards the violet, where the waves are short, than towards the red. Hence more wave-lengths fall on a given area in the infra-red than on the same area in the violet. If allowance is made for this, the maximum value of the radiant energy from the sun occurs near to the D lines. In fig. 127 the wave-length is given in decreasing order from left to right since as the temperature of a body rises the shortest radiation emitted has a smaller and smaller wave-length.

Lamps.—It will be noticed that the ordinary filament lamp loses a tremendous amount of energy in the form of heat which is all wasted since the lamp is used to emit light. The higher the temperature of a source of light the more efficient is that source. When a body is heated the particles in that body are set into more rapid vibration. The vibrations cause waves to be sent out through the surrounding ether, and the warmer the body gets the more waves it sends out and the shorter is the average wave-length. So the body first gives out the infra-red rays, then, in addition, visible red, orange, yellow, and so on until all the colours are represented and ultra-violet rays are being emitted. The body is then at a white heat.

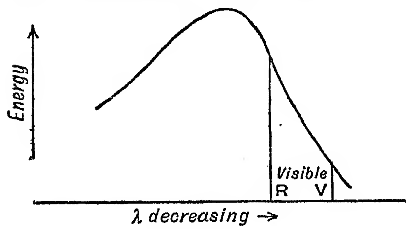


FIG. 128.

These changes can be represented by a graph of energy drawn against wave-length (fig. 128). Since the extreme wave-length becomes progressively less as the temperature rises the wave-length is shown decreasing from left to right. While the ordinate at any point shows the energy used in emitting light of that wave-length, it does not represent the total energy required, since radiation of that wave-length cannot be obtained without all other radiations (of this type) of greater wave-length. Thus the area beneath the curve represents the total energy required. So for a body at a white heat the energy usefully employed is represented by the area below the curve and bounded by the ordinates through R and V, while the total energy required is given by the whole area beneath the curve. This is a very inefficient method of obtaining light since more than 90 per cent. of the energy is lost in the form of heat, *i.e.* in infra-red radiation.

A more efficient lamp is the discharge tube in which an electric discharge is passed through a gas or vapour. This

causes the emission spectrum of that element to be given out. In most cases the gas or vapour used gives out lines in the visible and invisible parts of the spectrum, and it must be so chosen that as large a proportion of waves in the visible part as possible shall be obtained. Other factors naturally affect the choice of the gas, and neon gas and mercury vapour are most frequently used. In these cases greater efficiency is obtained than for the metal filament lamp. The latest discharge lamps (Osram) have an efficiency of nearly 40 lumens per watt, while the tungsten filament (gas-filled) lamp—previously the most efficient—has the much smaller efficiency of 15·5 lumens per watt.

For advertising, discharge lamps or tubes are now used considerably. The wattage is small so that they are economical in use. With neon as the gas red light is produced, while a mixture of neon and mercury gives blue light when contained in ordinary glass, but green light when contained in uranium glass.

Uses of Infra-red and Ultra-violet Radiations.—Since the advent of photographic plates sensitive to infra-red rays the science of photography has been further extended. As is pointed out in Chapter XV, the greater the wavelength the less is the scattering of light by small particles. So photographs taken using infra-red plates are very much clearer than those taken with ordinary plates. Further, since infra-red rays penetrate smoke and haze easily, objects a large distance away and not seen too clearly by the naked eye can be photographed.

In television the object is often illuminated by infra-red rays. The light reflected back falls on a photo-electric cell and sets up a current which can then be amplified. In this connection infra-red rays are more suitable than ultra-violet radiations, since the latter have a harmful effect on persons who are being televised.

There are many other applications of photo-electricity in conjunction with these invisible rays. If a beam of

ultra-violet light shining on a photo-electric cell is interrupted, the photo-electric current is changed and this change may be utilised to work an electrical relay and sound an alarm.

Other Invisible Rays.—In 1895 Röntgen discovered that some rays were emitted from a discharge tube and were capable of affecting photographic paper or causing fluorescence even if opaque black paper was placed between the tube and the photographic or fluorescent paper. These rays were called X-rays or Röntgen rays, and were caused by the sudden stoppage of the electrons emitted from the cathode when they hit the walls of the tube. In modern X-ray bulbs the cathode, C, is made concave, and the electrons are focussed on to a piece of metal, B, called the anticathode. This is connected to the anode, A, in order to facilitate the discharge. A cooling arrangement (not shown in the diagram) is included, since a great deal of heat is generated by the loss of kinetic energy when the electrons hit the anticathode. The X-rays are emitted from the anticathode and spread out in all directions. They have great powers of penetration, and will penetrate distances approximately proportional to the reciprocal of the density of the substance. Thus, paper is quite transparent to X-rays, while metals, of much greater density, are practically opaque. These rays are of great use in surgery since they penetrate through flesh far more easily than through bones. The power of penetration depends on the degree of exhaustion of the X-ray tube, and "hard" X-rays—those emitted from discharge tubes at low pressure—penetrate more than "soft" X-rays, from tubes in which the degree of exhaustion is not so great.

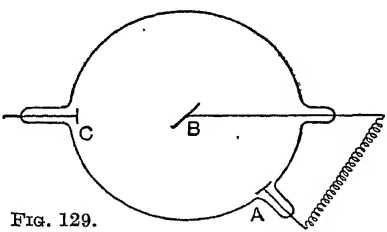


Fig. 129.

For many years the nature of X-rays was unknown, and

it was not until 1912 that Laue, with the help of Friedrich and Knipping, showed that these rays could be diffracted in the same manner as light. Instead of the artificial diffraction grating used for the latter, a crystal was used, the regularity of the spacing of the atoms giving the effect of a grating. This was the first step in showing that X-rays were of the same nature as light, and further experiments, notably those by W. H. and W. L. Bragg, have confirmed this and also given values for the wave-lengths of the rays. X-rays reflected by a crystal will interfere, since some are reflected at one crystal plane and other rays are reflected at other planes. For maximum reflection

$$n\lambda = 2d \sin \theta,$$

where d is the perpendicular distance between the planes, n is a whole number, λ is the wave-length of the X-rays, and θ is the angle between the wave front and the normal to the surface. By calculating d and measuring θ for spectra of various orders, λ has been determined and is of the order of one angstrom, varying according to the pressure in the discharge tube and the P.D. across the tube. The fact that the wave-length is so short accounts for the difficulty in reflecting and diffracting X-rays, for even the smoothest artificial surface would be too rough and irregular for reflecting these short rays, while it would be impossible to rule a diffraction grating with the lines close enough to diffract waves of such short length.

Somewhat similar in properties to X-rays are the γ rays of radium. They have great powers of penetration, affect a photographic plate and fluorescent paper and also cause ionisation in gases. It appears that γ rays are produced by the sudden stoppage of β particles inside a radio-active substance, just as the sudden stoppage of electrons causes X-rays. Using a modified form of Bragg's X-ray spectrometer, Rutherford found that γ rays had wave-lengths of the order of 10^{-2} angstroms and less.

Still shorter are the cosmic rays discovered and investigated by Millikan in America. These waves have great

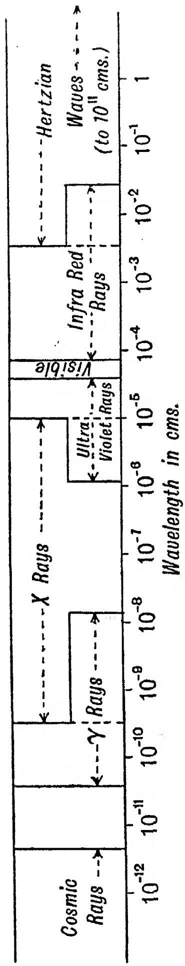


FIG. 130.

powers of penetration, and enter our atmosphere from other heavenly bodies. This means that energy is brought into the earth from outside agencies. This energy is possibly produced when four hydrogen nuclei combine to form a helium nucleus with a consequent loss in mass, causing a radiation of energy.

The wave-length of these rays is extremely small, of the order of 10^{-12} cm. Their penetrative powers are much greater than those of γ rays, even though some of the rays are absorbed in passing through the earth's atmosphere.

At the other end of the spectrum we have electro-magnetic waves (wireless waves) which are set up, for example, when an induction coil is worked in conjunction with an oscillatory circuit. With the ordinary valve oscillator it is possible to produce waves with a length as great as 10^{11} cm., while, using the spark transmitter similar to that used by Hertz in his original experiments in 1885, electric waves of length ·002 cm. have been obtained, although these are damped oscillations. Since heat waves with a length of 10^{-2} cm. have been obtained, it will be seen that the electric and heat waves overlap.

We thus have waves as short as 10^{-12} cm. and as long as 10^{11} cm., while in between lie the γ rays, X-rays, ultra-violet rays, visible light and infra-red rays, the only unknown portion being that between the γ rays and the cosmic rays. All these radiations travel with the same velocity through the ether, the

value of this velocity being almost exactly 3×10^{10} cm. per second.

On the wave-length chart (fig. 130) these rays are all shown, and the part occupied by the visible spectrum will be seen to be extremely small in comparison with the range of wave-lengths known. The wave-lengths are plotted in logarithmic fashion in centimetres.

EXAMPLES ON CHAPTER X

1. What are "infra-red" and "ultra-violet" rays? Describe how you would demonstrate the existence of these forms of radiation. (Lond. Inter.)

2. How would you investigate the laws of emission and absorption of infra-red radiation? (Lond. Inter.)

3. Describe experiments to show that the spectrum of a source of light, such as a carbon arc, extends beyond the visible part at both the red and violet ends, and explain carefully the methods adopted to observe the different regions of the spectrum.

Explain the principle of the Direct-Vision Spectroscope.

4. Write a short account of the distribution of energy in the spectrum of a full radiator maintained at such a temperature that it is emitting visible radiation. This account should show (*a*) how the spectrum may be produced, (*b*) how the energy distribution may be measured, and (*c*) the nature of the results to be expected. (N.)

5. Describe briefly how you would project a clear solar spectrum on to a screen, and draw a ray diagram to illustrate the arrangement of the apparatus.

How would you show that the spectrum extends in both directions beyond the visible region? (N.)

6. What are the principal points of resemblance and difference between radiant heat and visible light? How would you demonstrate them experimentally? (O. & C.)

7. Give a general account of the solar spectrum, and compare the physical properties of the visible and non-visible portions of it. (O. & C.)

8. Give a brief account of the phenomena of phosphorescence and fluorescence. How would you examine the nature of fluorescence? (Lond. Inter.)

9. What is meant by phosphorescence, and how did Becquerel observe it? (Lond. Inter.)

CHAPTER XI

THE RAINBOW: COLOUR

SMALL drops of water, whether in form of rain or of spray, refract and then reflect light so that by reason of the refraction the light is split up into its component colours. Sunlight, falling on raindrops, gives this effect, thus forming the rainbow. In every case the observer must have his back to the sun for a rainbow to be observed. The primary bow, formed by rays reflected once inside the raindrop, is the brightest bow, and is red on the outside and violet on the inside edge. The secondary bow, not so bright as the primary, is larger, and the colours in this case are in reverse order. Occasionally other bows—very faint ones—are seen inside the primary. These are caused by diffraction and are called supernumerary bows.

The Primary Bow.—Consider a ray of light PQ falling, at angle of incidence i, on a spherical drop of water of refractive index μ. Some of the light passes into the drop in the direction QR, making an angle r with the normal, and meets the outer surface of the drop at R where reflection occurs.

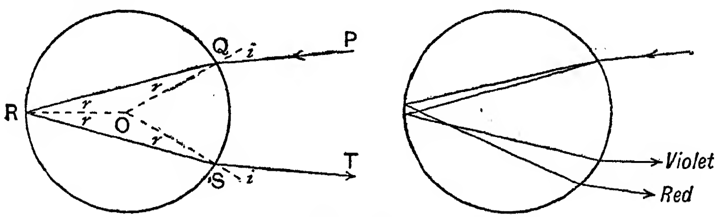


FIG. 131.

Only a fraction of the light is reflected, the remainder being transmitted. After this reflection the path of the ray is along RS until refraction back into air occurs, the

emergent ray being ST. It will be seen from the diagram that

$$O\hat{Q}R = O\hat{R}Q = O\hat{R}S = O\hat{S}R = r,$$

and that the angle of emergence must be i. Then the deviation produced is $(i - r)$ at the first refraction, $(180° - 2r)$ at the reflection, and $(i - r)$ at the second refraction. Thus the total deviation (D) is

$$180° + 2i - 4r.$$

The maximum value for D is obviously 180°, and occurs when the incidence is normal. The minimum value may be found by differentiation,

$$\frac{dD}{dr} = 2\frac{di}{dr} - 4.$$

But for D a minimum, $\frac{dD}{dr} = 0$, so that $\frac{di}{dr} = 2$.

Now

$$\sin i = \mu \sin r.$$

$$\therefore \cos i \frac{di}{dr} = \mu \cos r.$$

Hence

$$\begin{aligned} 2 \cos i &= \mu \cos r \\ &= \mu \sqrt{1 - \frac{\sin^2 i}{\mu^2}} \\ &= \sqrt{\mu^2 - \sin^2 i}. \end{aligned}$$

$$4(1 - \sin^2 i) = \mu^2 - \sin^2 i,$$

whence

$$\sin i = \sqrt{\frac{4 - \mu^2}{3}}.$$

For red light $\mu = 1{\cdot}329$, and for violet light $\mu = 1{\cdot}343$. Substitution of these values shows that for red light $i = 59{\cdot}65°$ and D is $137{\cdot}3°$, while for violet light $i = 58{\cdot}8°$ and D is $139{\cdot}2°$.

Now the light may be deviated by any value between 180° and 137·3°, but most of the light is reflected at angles such that the deviation is either a minimum or near to this value. This can be seen from the graph of angle of incidence against angle of deviation, where little change occurs in the deviation for a large change in the angle of incidence in the neighbourhood of 60°.

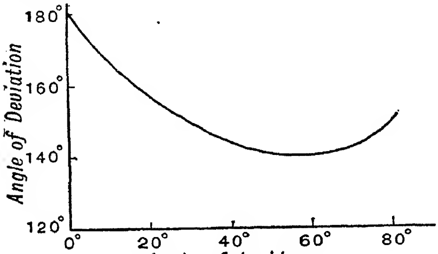


FIG. 132.

So for any raindrop illuminated by the sun, the resultant beam of light, formed after one internal reflection, will be most intense in a direction for which the deviation is 137·3° for red light and 139·2° for violet light. The drops which reflect out this light lie on the arcs of concentric circles—the centre being on the line from the sun passing through the observer O. A series of cones is formed with the circles as bases and the observer as apex. For red light the semi-vertical angle of the cone is $\hat{COE}$ and

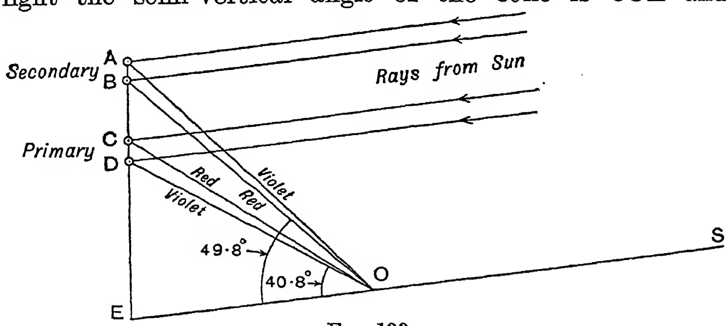


FIG. 133.

equals 42·7° (180°–137·3°), while for violet the semi-vertical angle is $\hat{DOE}$ equalling 40·8°. Since large parts

of the circles pass below ground level, in general, the reflected light seen is in the form of a bow with the red on the outside.

The Secondary Bow.—For certain angles of incidence the light is reflected twice within the raindrop before it is emitted in the direction of the observer. Since at each reflection the deviation is increased by $(180°-2r)$, where r is the angle between the normal and the ray inside the drop, the total deviation for a ray twice reflected is

$$360° + 2i - 6r.$$

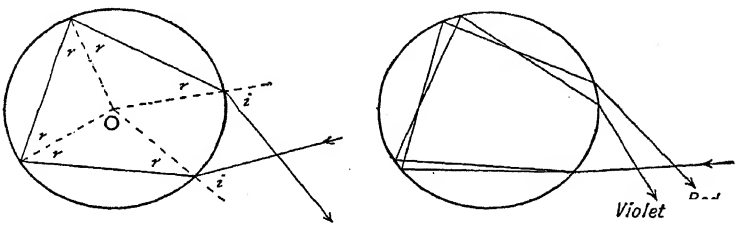


Fig. 134.

Proceeding as before it is found that for minimum deviation the angle of incidence is given by

$$\sin i = \sqrt{\frac{9-\mu^2}{8}},$$

so that the angle of minimum deviation is 229·8° for red and 233·6° for violet light. It will be noticed that in this case the emergent rays cross the incident rays from the sun, the angles between these rays being 49·8° (229·8°−180°) for red and 53·6° for violet. Consequently in this case another bow is formed and the order of the colours is reversed, violet now being on the outside. This bow is fainter than the primary since light is lost, to the observer, at each reflection. The formation of this bow is similar to that of the primary, and is shown by fig. 133.

Colour

In studying the subject of colour a distinction must be made between coloured lights and pigments. If two coloured lights are superimposed, the colour produced is not the same as that obtained when two pigments, of the same colours as the lights, are mixed. There are two reasons for this difference: (1) pigmental colours are impure while coloured lights are pure; (2) when coloured lights are mixed the resultant colour is the sum of the constituent colours, so that the process is one of addition. With pigments, each absorbs certain colours from white light, and the resultant colour, after mixing, is the one which is common to all the pigments mixed; this is thus a process of subtraction.

The Colour Triangle.—It is found that almost all colours may be obtained by suitable mixture of three colours—red, green and blue (containing violet). These three are termed the primary colours, and if mixed themselves they produce white light. This does not mean that white light will emerge when it is passed successively through filters of red, green and blue. For a red filter only lets through red light which would not pass through a green or a blue filter. In order to mix coloured lights, separate sources of white light are used, one colour filter being placed in front of each source, and the beams are made to overlap on a screen. Then it is found that red and green produce yellow, red and blue yield magenta, and green and blue give peacock blue. The effect is represented by an equilateral triangle. The primary colours are at the corners, and the three colours obtained on mixing the primaries are at the centres of the sides. The three colours—magenta, yellow and peacock blue—are called the secondary colours. In the centre of the triangle is white, which is produced by mixing (*a*) the three primary colours, or (*b*) the three secondary colours, or (*c*) a primary colour with the secondary opposite to it in the triangle.

Other colours are obtained by mixing these colours in unequal proportions. Thus, if we pass along the side of the triangle from red to green we reach orange before yellow. The orange is the result of mixing red and green colours, the intensity of the red being about twice that of the green. Since the centre of the triangle is nearer to the mid-points of the sides than to the corners in the ratio 2 to 1, it follows that white light produced by mixing a primary and a secondary colour will only be obtained when the intensity of the latter is twice that of the former.

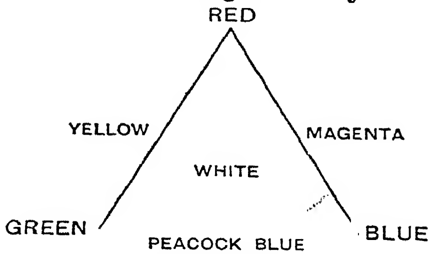


FIG. 135.

Mixing Coloured Lights.—The purest colours are obtained by means of the spectrometer. It will be remembered that Newton, after forming a spectrum, passed each colour through a prism and showed that no additional colour was produced. Also it has already been mentioned that white light is produced if the spectrum colours are combined either by means of a convex lens or by seven small plane mirrors, arranged in the form of an arc. Variations of these methods may be used for examining the effects obtained when the primary colours are combined. A spectrum is formed on a screen which contains slits so cut that the primary colours only pass through. These three colours can then be combined by use of a convex lens. Or one of the slits may be covered so that the result of combining two primary colours may be observed. Using the mirrors it is necessary to remove certain of the mirrors so that only the desired colours will be reflected on to the screen.

The colour disc, on which are equal sectors of red, green and blue, gives the same effect when rotated rapidly. Best

results are obtained when the disc is well illuminated. The colours merge and produce grey (which is a mixture of white and black) when weakly illuminated, but white when the illumination is more powerful. If only two of the colours are used, the resulting secondary colour is obtained.

These methods are not very satisfactory in practice and it is usually better to use three sources of white light with the respective colour filter in front of each.

Mixing Pigments.—It has already been stated that the process of mixing pigments is one of subtraction of colour. A pigment absorbs certain of the colours of the white light which may fall on it, so that it then appears coloured itself. But a pigment contains other colours in addition to the one which predominates. Therefore, if two pigments are mixed, the colour of the mixture is the colour which occurs in each pigment. The effect can be seen easily by using filters of the primary and secondary colours. If red and green filters are placed together no light at all passes through since these colours are pure and no colour is common to them. But if filters of magenta and yellow are placed together and held in the path of white light, then the emergent light is red, since red is common to each of these colours. Similarly with magenta and peacock blue, blue results, while with yellow and peacock blue the result is green. This explains why green is produced when yellow and blue pigments are mixed. The yellow contains red and green, while the blue contains green. As green is the only colour common to both pigments the mixture appears green.

The Pigments Triangle.—Artists usually regard the three primary colours as red, yellow and blue. As far as pigments are concerned this choice of primary colours is satisfactory, but it must not be applied to coloured lights. The primary colours (for lights) should be pure so that none of them can be formed by mixing other coloured lights. Strictly, the primary colours are the only pure colours.

Then it becomes obvious that green must be a primary colour since it is impossible to produce green by mixing together any coloured lights. But light which is apparently yellow can be obtained by mixing orange and green lights, so that yellow cannot be considered as a primary colour.

As far as pigments are concerned, however, it is convenient to regard yellow as a primary instead of green. We can then construct a triangle for pigments. This resembles the colour triangle, but if the primaries are mixed, black results. The same result is obtained if all the pigments

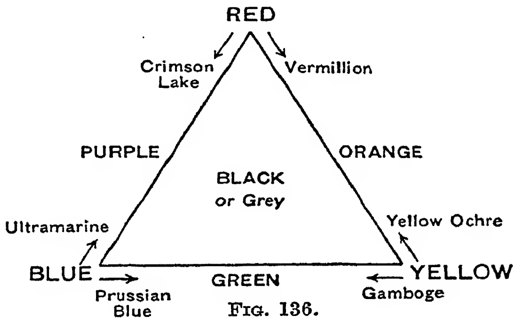


FIG. 136.

which occur in this triangle are mixed. The great difference between the colour triangle and the pigments triangle is that the former shows the results obtained by addition of colours, while the latter shows the results of subtracting colours.

The triangles illustrate the difference in meaning between the words "tint" and "shade." A tint is produced when white is added to a colour. A shade results when white is subtracted from a colour (or black is added to the colour).

Owing to the impurities in colour of a pigment, the pigment assumes different tints as the colour of the incident light changes. Thus a yellow pigment may appear red under the influence of red light, and green when illuminated by a green coloured light.

Since most artificial lamps give out a higher percentage

of red and orange light than does natural white light, the colour of an article does not appear the same when illuminated by the lamp as it does in daylight.

Complementary Colours.—Two colours which produce white when mixed are termed complementary colours. From the colour triangle it is at once seen that yellow and blue, or green and magenta, or red and peacock blue are complementary colours. There are, however, many more combinations for producing white light. Any two colours whose positions on the triangle can be joined by a straight line passing through the centre of the triangle yield white light when mixed in the correct proportions. As an example, greenish-yellow is complementary with violet (lying between blue and magenta).

Simple experiments on complementary colours may be carried out by aid of the small plane mirrors (or a single cylindrical mirror) on which a spectrum is formed. By obscuring the mirror on which red light falls, the combined light then is greenish-blue. If the green is cut out, magenta results, while elimination of the blue and violet produces an orange shade. Similar results may be obtained by using a convex lens to combine the light. If one colour is screened off, the complementary colour is obtained on the screen.

Helmholtz found that not only are colours which contain ranges of wave-lengths complementary, but that definite wave-lengths could be combined to produce white light. Thus a large number of pairs of wave-lengths are complementary.

Colour Vision.—It is supposed that in the eye there are three different kinds of nerves—one being sensitive mostly to red light, another to green light and the third to violet light. Most wave-lengths excite two of the three sensations to different extents, and certain wave-lengths, between about 5400 A.U. and 4300 A.U., excite all three nerves. White light results when each sensation is caused to the same extent. Thus for any two colours to be com-

plementary they must excite each nerve equally. When red light falls on the eye only the red sensation is produced. Yellow light excites both the red and the green sensations to similar extents, blue light excites all three sensations but not to the same extent, while violet light affects the violet and the red sensations, the affect on the red, however, being very small.

This theory explains why it is that a white screen, when viewed immediately after the eye has been focussed on a coloured object, appears to be coloured itself. The apparent colouring of the white screen is always complementary to the colour of the object first viewed. Certain of the nerves in the eye become fatigued by continuous reception of the light from the coloured object, so that when white light falls on the eye the other nerves are affected to a comparatively greater extent and the sensation of colour results. Thus after the eye has viewed a red object for some moments, a white screen seen immediately afterwards appears to be greenish-blue.

A similar sensation results when the eye views a sheet of white paper on which several black lines are drawn. The nerves on which the white light falls become fatigued, so that if a white screen is viewed immediately, it appears to be a little dull (or grey) with white lines drawn across it. Equal quantities of light fall on all the nerves in the eye, but those which have been previously viewing the black lines are not fatigued and are thus momentarily affected to a greater extent than the others.

Similar changes in the apparent colour of an object occur when the object is placed on a background which is coloured. A green object excites both the red and the green sensations in the eye so that if a small green object is placed on a red background, the nerves affected by red become fatigued and the object appears to be coloured a stronger green than it actually is. In the same way a small white object on a red background appears to be green.

If one of the sets of nerves in the eye is insensitive, the

eye does not interpret colours correctly. This gives rise to colour-blindness. In most cases it is the red sensation which is lacking, so that to most colour-blind persons the spectrum seems to be shortened at the red end, and in the hydrogen spectrum no red line is visible. Colour-blindness is sometimes referred to as Daltonism since Dalton, the chemist, was colour-blind and recorded his impressions on this subject at a time when few investigations of it had been made.

EXAMPLES ON CHAPTER XI

1. To what is colour of bodies, as ordinarily seen, to be attributed? (Lond. Inter.)

2. A pure spectrum is formed on a screen which is painted green. What colours will be seen on the screen?

3. What do you understand by the terms tint, shade, and purity of colour?

Explain why yellow and blue pigments when mixed do not yield the same result as that obtained on mixing yellow and blue lights.

4. Write a short account on complementary colours, with reference to the colour triangle.

5. Describe the formation of the primary and secondary rainbows.

6. Describe how you would show experimentally that white light is composed of a mixture of colours.

Show also how these colours may be recombined to form white light. (O. & C.)

CHAPTER XII

THE VELOCITY OF LIGHT

THE velocity of light is one of the most important of the universal constants. It is the highest speed attainable, and is the speed with which cosmic rays, X-rays and wireless waves travel. This velocity is involved in the ratio of the dimensions of electrical quantities when measured in electrostatic units to the dimensions in electromagnetic units. It is therefore important that the velocity of light should be known accurately. Early investigators found it impossible to get any trustworthy results owing to the very great velocity of light.

Römer's Method.—Römer observed that the time between successive eclipses of one of Jupiter's moons was not always the same. He noticed that the time was greater when the earth was receding from Jupiter. The reason for this difference in the times is that when the earth is moving away from Jupiter the light has a greater distance to travel so that the eclipses appear late. By finding how much later the eclipse is when the earth and Jupiter are in opposition than when these planets are in conjunction, Römer obtained a value for the velocity of light in 1676.

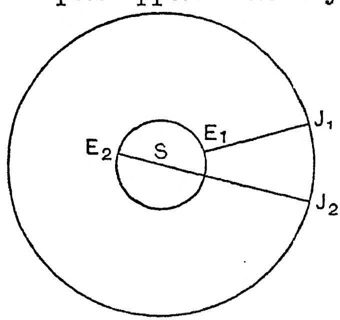


FIG. 137.

Let E_1, J_1 be the positions of the earth and Jupiter when in conjunction, S being the position of the sun. Then if one of Jupiter's moons is eclipsed, this eclipse is noticed on the earth J_1E_1/V seconds later, where V is the velocity

of light. Nearly seven months later another eclipse of the same moon is observed, the earth and Jupiter now being in opposition, E_2 and J_2. This eclipse is seen on the earth J_2E_2/V seconds after its occurrence. Suppose that the actual period between successive eclipses of this moon is T seconds, and that n eclipses occur as Jupiter moves from J_1 to J_2, then the eclipse at J_2 is seen $nT + \frac{J_2E_2 - J_1E_1}{V}$ seconds after the eclipse at J_1. By finding this time and knowing n and T, a value for V is obtained in terms of the distance $(J_2E_2 - J_1E_1)$. But this distance is the diameter of the earth's orbit and so may be found. Thus a value for the velocity of light can be obtained.

This method is not extremely accurate, but it is important since it established the fact that light travels with a finite velocity.

Bradley's Aberration Method.—In 1725 Bradley observed that certain fixed stars appeared to have a small motion in space. This he found was due to aberration of light caused by the motion of the earth. Let S be the position of a star, and let $S\hat{E}H$ be the true angle of elevation of the star, *i.e.* the angle of elevation which would be observed if the earth was at rest. Let V be the velocity of light and v be the velocity of the earth in the direction EH. Now while light from S travels towards E, the earth is moving, so that this light is received by an observer who was at F when the light left S, but who is now at E.

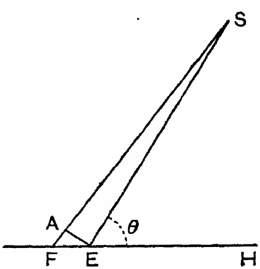


FIG. 138.

Thus $S\hat{F}H$ is the apparent angle of elevation of S. Now

$$\frac{FE}{SE} = \frac{v}{V},$$

showing that FE is small compared with SE. Hence if AE is perpendicular to SF we have

$$AE = EF \sin \theta$$

and

$$AE = SE \sin \delta,$$

where

$$\theta = S\hat{E}H \quad \text{and} \quad \delta = E\hat{S}F.$$

Thus

$$FE \sin \theta = SE \sin \delta$$

or

$$v \sin \theta = V \sin \delta,$$

which may be written as

$$V = \frac{v \sin \theta}{\ }$$

since δ is a small angle.

By measuring the quantities on the right-hand side of this expression Bradley determined the velocity of light, and confirmed the result already obtained by Römer.

Fizeau's Method.—Early attempts to measure the velocity of light by terrestrial methods were unsuccessful, and the first method to meet with success was carried out by Fizeau in 1849. He arranged that light from a source S should be rendered convergent by means of an achromatic lens L, so that an image of S should be formed at I

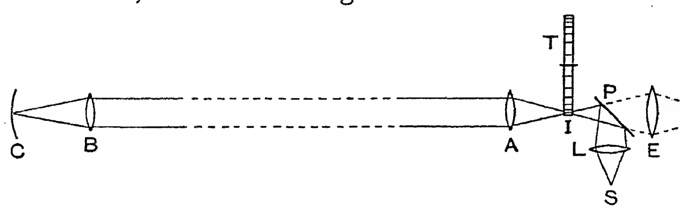


Fig. 139.

after the light had been reflected at a semi-silvered glass plate P. The light was then made parallel by a lens A and passed over a distance of about five miles before

reaching another lens B. This lens brought the light to a focus on a concave mirror C, the radius of curvature of which equalled CB. Thus after reflection the light travelled back to form another image at I. This image was viewed through P by means of an eyepiece E. A toothed wheel T was arranged to revolve so that when one of the teeth was at I no light could pass. The speed of rotation of T was adjusted so that no image was seen. When this happened the light from S which passed through a gap between two of the teeth returned from C just as a tooth moved into position at I.

Fizeau observed that as the speed of rotation was gradually increased, images alternately appeared and disappeared, as the returning light met either a gap or a tooth.

Let the distance $IC = d$, and let the angular velocity of T be w radians per second for the first disappearance, *i.e.* the smallest speed of the wheel for which no image is seen. This occurs when the light from S passes through a gap and on returning is intercepted by a tooth adjacent to this gap. If the wheel has n teeth and the width of a tooth equals the width of a gap, then for the first disappearance T must have turned through an angle π/n radians. The time taken for this is π/nw seconds. But in this time light travels from I to C and back.

Hence

$$V = \frac{\text{Distance moved}}{\text{Time taken}}$$

$$= \frac{2d}{\pi/nw}$$

$$= \frac{2dnw}{\pi},$$

where V is the velocity of light. The value of V obtained by Fizeau was $3{\cdot}15 \times 10^{10}$ cm. per second.

It is possible to criticise this method in several instances.

For example, when light from S is intercepted by a tooth, some of the light is reflected back into the eyepiece and causes the field to be illuminated. This disadvantage was overcome by Young and Forbes, who repeated the experiment in 1880. They bevelled the teeth so that this light was reflected to the sides and not into the eyepiece. Another disadvantage overcome by these experimenters was the loss of light at the semi-silvered plate P. Of the incident light from S only half was reflected at P, and of the light which returned from C only half was transmitted into the eyepiece. Consequently the image was rather faint. Young and Forbes used a silvered glass plate with a small area unsilvered near to the centre. A third difficulty in the original experiment was to tell the exact speed of the wheel for which the light returning from C was stopped by the middle of the tooth. This difficulty was overcome by Cornu in 1874. He measured the speed of rotation when the brightness of the image decreased to a certain value, and again found the speed when the image, after disappearing, reached the same degree of brightness as before.

Cornu experimented over a distance of nearly 15 miles, the mean of his results for the velocity of light being $3{\cdot}004 \times 10^{10}$ cm. per second.

Young and Forbes slightly changed the method for their experiment. The combination of B and C was arranged to reflect back only half of the parallel beam from A. The other part of the light passed on to an exactly similar combination of lens and concave mirror farther away. This light on returning to I had travelled a greater distance than that reflected at C. The velocity of light was determined by finding the speed of rotation of the wheel for which the two images formed at I were equally bright.

Foucault's Rotating Mirror Method.—Ten years after finding the velocity of light by the toothed wheel method Fizeau—now in conjunction with Foucault—set out to determine the velocity by a rotating mirror method. The

method is usually attributed to Foucault because the partnership was dissolved before the experimental details were completed, and Foucault then proceeded to carry out the experiment himself.

Light from the source S is made slightly convergent by means of an achromatic lens L, is reflected from a plane mirror M, and brought to a focus on a concave mirror C. The plane mirror is capable of rotation at a high speed about an axis perpendicular to the plane of the paper. The centre of curvature of C lies on this axis, ensuring that the beam reflected from C to M will travel over exactly the same path as the incident beam. The reflected beam

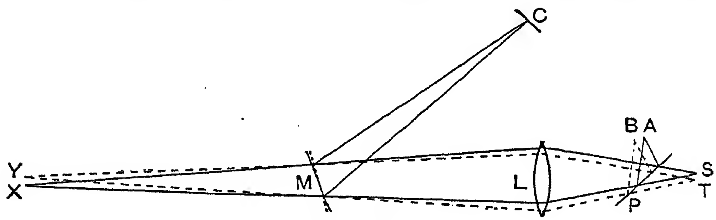


FIG. 140.

thus forms an image at S. For convenience of observation a semi-silvered glass plate P is introduced, so that the final image is seen through an eyepiece at A.

Now suppose that M is rotated rapidly. In the short interval of time necessary for the light to travel from M to C and back, M rotates through a small angle and the light reflected from M travels back along a slightly different path. Thus the image is seen at B instead of at A. This distance AB is very small, but it can be measured by means of an eyepiece. The only other measurements required are certain distances and the speed of rotation of the mirror.

Let SL $=a$, LM $=b$, and MC $=d$, and w radians per second be the angular velocity of M. Also let the time taken for the light to travel from M to C and back to M be t seconds.

When M is at rest the light returning to L appears to diverge from X, where X is the image of C formed by M. When M has moved through an angle wt, the returning rays appear to come from Y.

Since a reflected ray is turned through twice the angle through which the mirror turns, XY must subtend an angle $2wt$ at M, for M itself turns through wt.

Hence

$$XY = 2wtd,$$

since

$$XM = MC = d.$$

Now AB, or ST, is the image of XY formed by the lens L.

$$\therefore \frac{AB}{XY} = \frac{a}{b+d},$$

or

$$AB = \frac{2wtda}{b+d}.$$

Writing V for the velocity of light we have

$$V = \frac{2d}{t},$$

so that

$$AB = \frac{4wd^2a}{V(b+d)},$$

or

$$V = \frac{4wd^2a}{x(b+d)},$$

where $x = AB$.

A great drawback to this method is that the distance MC must be small. At the same time this is partly an advantage in that the experiment can be performed in a laboratory. As M rotates, it reflects light in all directions in the plane of the paper. So if δ is the width of C, only during a fraction $\frac{\delta}{4\pi d}$ of a revolution does light fall on C.

This means that the brightness of the image at B is this fraction of the brightness of the image seen when M is at rest. If MC is increased then the brightness of the image is reduced. Michelson repeated this experiment in 1882, and to a large extent overcame this difficulty by placing the achromatic lens L between M and C. The diameter of L was fairly large, so that light was made to fall on C for a much larger fraction of a revolution of M than before. By this arrangement it was possible to increase the distance CM to 600 metres without any loss in brightness of the image.

As a consequence the displacement of the image was increased to 133 mm. In Foucault's experiment the displacement was 0·7 mm. for a distance of 20 metres between M and C, the value for V being $2 \cdot 98 \times 10^{10}$ cm. per second.

It should be pointed out here that the laws of reflection —which are proved for stationary mirrors—are assumed, in this experiment, to be true, although the mirror M is rotating.

Foucault repeated his experiment with a long tube of water placed between M and C. He found that the displacement of the image was greater than before, and so proved that the velocity of light is greater in air than in water. This result was of the greatest importance, since the corpuscular theory of light required a higher velocity the denser the medium. Foucault did not actually determine the velocity in water, but Michelson, using a similar apparatus, found that the ratio of the velocity in air to that in water was 1·330. Since the refractive index of water is 4/3 the result is in good agreement with that expected on the wave theory of light.

Michelson's (*Mount Wilson*) *Method.*—None of the methods described as yet is extremely accurate, and the most reliable value for the velocity of light is the one determined by Michelson. Light from a source S is reflected at one of the faces of an octagonal mirror M, and by a system of mirrors (or right-angled prisms arranged

to reflect the light) it falls on a concave mirror C which produces a parallel beam. This beam travels to another concave mirror D, at the focus of which is a small plane mirror P. The light is thus made to return along a similar

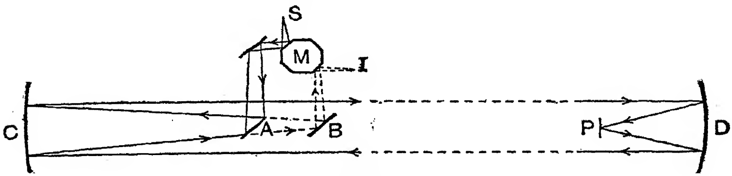


FIG. 141.

path, and on returning to C it is reflected through A, which is semi-silvered, to a small mirror B. Here reflection occurs, and the beam falls on another face of the octagonal mirror before being brought to a focus at I where it may be conveniently observed.

The octagonal mirror is rotated and its speed gradually increased until the image I is in exactly the same position as the one it occupied when M was at rest. For this to occur M must turn through one-eighth of a revolution while the light travels to P and back. Thus, instead of the returning light hitting the same face as before, it now hits the adjacent face. For greater speeds of rotation of M the same effect occurs, so that while the light travels to P and back, M turns through one-eighth or two-eighths or three-eighths . . . of a revolution.

Taking the first of these cases, we find that M must have turned through an angle $2\pi/8$ radians while the light travelled to P and back. Hence

$$\frac{2\pi}{8w}=\frac{2d}{\mathrm{V}}$$

or

$$\mathrm{V}={}^{8dw}$$

where w is the angular velocity of M in radians per second

and d is the distance the light travels from S to P, V being the velocity of light.

The distance d was approximately 22 miles between Mount Wilson and S. Antonia. Great accuracy was obtained in this experiment, and the value obtained for the velocity of light, $2{\cdot}9983 \times 10^{10}$ centimetres per second, is considered to be very reliable.

EXAMPLES ON CHAPTER XII

1. Describe how it has been proved that the velocity of light is less in water than in air. Explain the theoretical importance of this result. (Lond. Inter.)

2. How can it be shown by direct experiment that light travels slower in water than in air?

What evidence have we that blue rays have the same velocity as red rays through free space, but a smaller velocity in glass? (Lond. Inter.)

3. Give an account of some method by which the velocity of light has been determined. How has it been shown that the velocity of a light signal in water is less than its velocity in air? (O. & C.)

4. Give a brief account of not more than three of the experimental methods which have been employed to determine the velocity of light. (N.)

5. Explain fully how the velocity of light has been determined from astronomical observations.

Calculate the ratio of the longest to the shortest interval between successive eclipses of one of Jupiter's moons, being given that the orbital velocity of the earth is v cm./sec. and the velocity of light is V cm./sec. (Lond. H.S.C.)

6. Draw a diagram showing the arrangement of the apparatus and the path of the rays of light in Fizeau's toothed wheel method for measuring the velocity of light.

What are the chief difficulties met with in carrying out the experiment?

If the wheel has 150 teeth and 150 spaces of equal width, and its distance from the mirror be 12 km., at what speed, in revolutions per minute, will the first eclipse occur? (Velocity of light $= 3 \times 10^{10}$ cm. per sec.) (N.)

7. Give a critical account of Fizeau's method of determining the velocity of light. Point out the disadvantages of the first experiment, and explain how they were overcome by subsequent experimenters.

8. A beam of light is reflected by a rotating mirror on to a fixed mirror, which sends it back to the rotating mirror from which it is again reflected, and then makes an angle of 18° with its original direction. The distance between the two mirrors is 10^6 cm., and the rotating mirror is making 375 revolutions per second. Calculate the velocity of the light. (Lond. Inter.)

9. Describe the revolving mirror method of measuring the velocity of light, and explain how the value would be deduced from the observations.

Using this method the velocity of light in water was shown to be less than the velocity of light in air. Of what importance is this result? (Lond. H.S.C.)

10. Describe the revolving mirror method of determining the velocity of light, and show how the result is calculated from the observations made. (O. & C.)

CHAPTER XIII

THEORIES OF LIGHT

The Corpuscular Theory.—Before the time of Newton it had been suggested that light rays consisted of particles or corpuscles moving with a very high velocity. When these particles reached the eye they caused the sensation of vision. It is easy to see that on this theory the rectilinear propagation is at once explained, for the high speed particles move in straight lines. Newton took up this theory and elaborated it in order to explain reflection and refraction.

In the case of reflection he assumed that the particles experienced a force of repulsion when they neared a reflecting surface, and so rebounded with no change in their velocity (v). Since the component velocity parallel to the reflecting surface is unchanged, $v \sin i = v \sin r$,

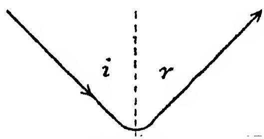


FIG. 142.

where i and r are the angles of incidence and reflection respectively. Hence $i = r$, and since the paths of the particle before and after reflection are in the same plane as the normal at the point of incidence, the laws of reflection are verified.

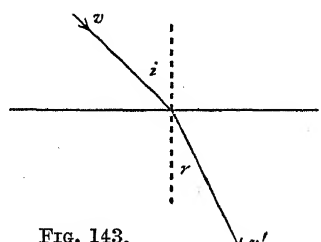


FIG. 143.

If, however, the particle approaching a surface is in a different condition it may be attracted to the surface and so pass into this other medium. In this case, taking $\mu > 1$ for the substance, the path of the particle is bent towards the normal as it passes from air to this

medium. If i and r are the angles of incidence and refraction, and v and v' are the velocities in air and the medium

$$v \sin i = v' \sin r$$

since the component velocities parallel to the boundary face are the same.

Thus $\dfrac{\sin i}{\sin r} = \dfrac{v'}{v} =$ a constant for the two media.

This result is of great importance, since it not only agrees with Snell's Law of Refraction, but also proves that the velocity of light is greater the denser (optically) the medium through which it travels.

Another case was considered by Newton—that of simultaneous reflection and refraction. To explain this, he assumed that the particles had "fits," so that some were in a state or condition favourable to reflection, and others in a condition suitable for transmission.

No explanation of interference, diffraction or polarisation of light was attempted, for indeed little was known about these phenomena at Newton's time.

The Wave Theory.—A rival theory was advanced in 1679 by Huygens, who postulated that light travelled in the form of waves.

A difficulty is met right at the start, for according to the wave theory light should travel round obstacles as sound does. Consequently the theory does not, at first sight, explain the rectilinear propagation of light, and this was one of the reasons which led Newton to reject the wave theory.

Just as water waves are set up when a stone is dropped into water, so we imagine that light waves are set up by the vibrations of electrons in incandescent substances. But the waves need a medium to travel through, and since light travels through space, it is necessary for that space to have the property of transmitting waves. The medium is called the ether, and it must possess elasticity (*i.e.* be able to withstand a stress), as well as density or inertia.

The vibrations in the wave leaving a point source are assumed to be in simple harmonic motion (S.H.M.). This is a special form of vibration, and is defined as the projection of uniform circular motion. Thus if P moves round a circle of radius a, with uniform angular velocity w, its projection N on any diameter AB moves in simple harmonic motion. If we imagine P to start from C, where $C\hat{O}A = 90°$, then after a time t, $P\hat{O}C = wt$, and the displacement of $N = ON = a \sin wt$. So writing y for the displacement we have $y = a \sin wt$.

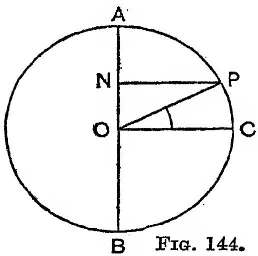


Fig. 144.

If T is the time for a complete rotation of P, then P describes an angle 2π in T secs., and hence $wT = 2\pi$. So $y = a \sin \frac{2\pi t}{T}$, which is often written in the form $y = a \sin 2\pi nt$, where n is the frequency of vibration or the number of complete waves emitted per second.

If P does not start at C when we commence timing, a phase angle ϵ is introduced, and the general expression is $y = a \sin (2\pi nt + \epsilon)$. This means that when $t = 0$, $y = a \sin \epsilon$.

The *phase* of a particle moving in S.H.M. is the fraction of the period which has elapsed since the particle moved past its initial position. The phase angle (ϵ) is thus the angle between OP (wherever the initial position of P may be) and OC measured in the direction in which P moves. The maximum displacement possible is a, which is called the *amplitude* of the motion. It is thus possible to re-

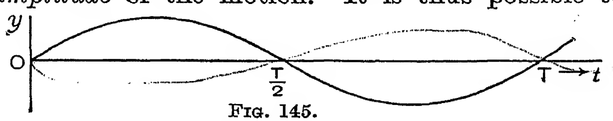


Fig. 145.

present the propagation of light waves by a sine curve. The distance between consecutive particles which have the

same displacement and which are moving in the same direction is called the *wave-length*. It is obvious that if n is the frequency and λ is the wave-length, $V = n\lambda$, where V is the velocity of the wave.

Relationship between Intensity and Amplitude.—In wave motion the energy is taken along with the wave, the medium being in the same condition after the wave has passed as it was before. This energy is proportional to the square of the amplitude of the motion.

The energy of N (fig. 144), moving in simple harmonic motion, is partly potential and partly kinetic, but the total is always the same. The proportions of these forms of energy vary, and when N is at O the energy is all kinetic.

$$\therefore \quad \text{Energy} = \tfrac{1}{2}\rho a^2 w^2 \text{ per c.c.}$$

where ρ is the density.

But the intensity of illumination I depends on the energy received. Thus

$$I \propto a^2.$$

Also, since

$$I \propto \frac{1}{(\text{distance})^2},$$

$$\text{amplitude} \propto \frac{1}{\text{distance}}.$$

Huygens' Explanation of Rectilinear Propagation.—In order to overcome the initial difficulty of explaining rectilinear propagation on the wave theory, Huygens' assumed that each point on the wave front could be treated as a source of secondary waves. If ABCD (fig. 146) is part of a spherical wave front, each point on ABCD gives rise to a spherical wavelet which after a time t will have travelled a distance vt, where v is the velocity of light in the medium. The disturbance originating at B will have spread to B′, that from C will have reached C′, and so on. Since these disturbances are taking place simultaneously the wavelets

all overlap to give a resultant wave front XY. The parts of the wavelets which overlap are moving in different

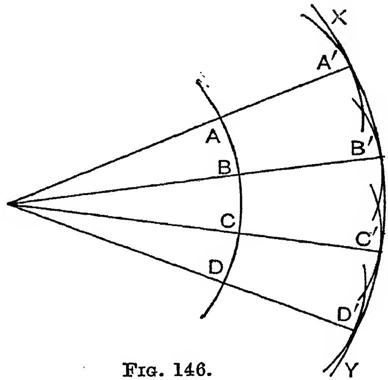


Fig. 146.

directions, and the components at right angles to the direction of propagation at any point cancel out.

In a similar manner the principle may be applied to plane waves or converging waves as shown.

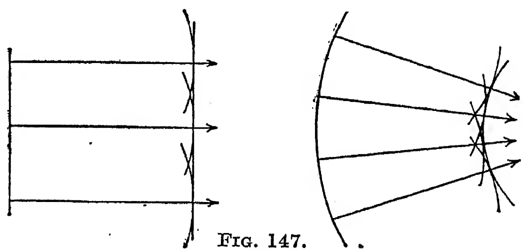


Fig. 147.

If a curved wave diverging from O (fig. 148) meets an obstacle in which there is a small aperture S, the resultant wave is diverging but has its centre at S instead of O, the original centre of the disturbance.

There is still a difficulty, however, for the secondary waves emitted by points near to edges of the initial wave are curved, and so show that the light is not travelling

entirely in straight lines, but bends round corners. An explanation was later forthcoming from Fresnel, who

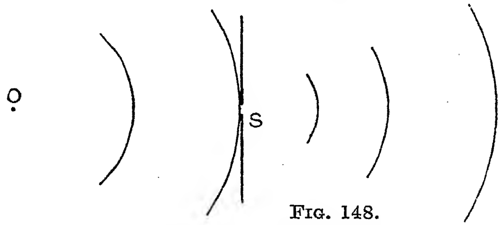


FIG. 148.

showed that this departure from the rectilinear propagation depended on the wave-length and was extremely small in the case of light where the waves are very short.

Although light is said to travel in rays—on which supposition almost the whole of geometrical optics is based—it is quite impossible to obtain a ray of light. If light is passed through several small circular apertures in succession, the emergent light is found to spread out almost as though the source of light is at the last of the apertures. Thus a ray of light is used merely for convenience, and should be understood to mean the direction in which some point on the wave front is travelling.

Fresnel's Half-Period Zones.—Let OP be the normal to a

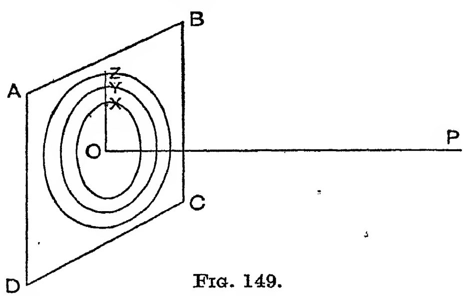


FIG. 149.

wave front ABCD, and let OP $= r$. With centre P and radii

$$r + \frac{\lambda}{2}, \quad r + \frac{2\lambda}{2}, \quad \ldots \text{ etc.,}$$

draw spheres cutting ABCD in circles of radii a_1, a_2, etc. The areas enclosed are called half-period zones. Let OZ be a line cutting the circles at X, Y and Z. Now the secondary waves emitted from O travel a distance r to reach P, while those from X travel $r+\frac{\lambda}{2}$. There is thus a phase difference of $\frac{\lambda}{2}$ between these waves, and similarly a phase difference of $\frac{\lambda}{2}$ exists between waves from X and those from Y. So for each point in the first ring there is a point in the next ring such that the waves sent out reach P exactly out of phase. Thus the resultant intensity at P can be written as

$$I=s_1-s_2+s_3-s_4+\ldots,$$

where $s_1, s_2, s_3 \ldots$ represent the contributions of each zone.

Now the magnitudes of s_1, s_2, $s_3 \ldots$ depend on

(1) the area of the zone,
(2) the obliquity of the direction to P and the distance to P.

Now the areas can be shown to be equal if we neglect λ^2, as λ is small compared with r.

For

$$\begin{aligned} a_1^2 &= PX^2-PO^2 \\ &=(r+\lambda/2)^2-r^2 \\ &=r\lambda \ldots \text{ neglecting } \lambda^2/4, \end{aligned}$$

so that the area of the first ring

$$=\pi a_1^2=\pi r\lambda.$$

Similarly, the area of the second ring

$$=2\pi r\lambda-\pi r\lambda=\pi r\lambda,$$

and the area of each ring $=\pi r\lambda$.

Now the obliquity and distance to P increase as we

move along OX. Hence the numerical values of s_1, s_2, s_3, . . . gradually decrease, although approximately

$$s_2=\frac{s_1+s_3}{2}, \text{ etc.}$$

$$\begin{aligned}\therefore\ \mathrm{I}&=s_1-s_2+s_3-s_4+\ \ldots\\ &=\frac{s_1}{2}+\left(\frac{s_1}{2}+\frac{s_3}{2}-s_2\right)+\left(\frac{s_3}{2}+\frac{s_5}{2}-s_4\right)+\ \ldots\\ &=\frac{s_1}{2}.\end{aligned}$$

So the resultant intensity at P due to the waves from ABCD is half the intensity caused by the waves from the first half-period zone.

If a circle of radius $\sqrt{\frac{r\lambda}{2}}$ is described about O the area is half that of the first zone, and the resultant intensity at P may be considered as due to this tiny area, all the light from other points on the wave front interfering and so having no resultant effect at P. Now OP is a ray of light, and so it follows that since O can be at any distance from P the light reaching P must come from O or the small area round it. Thus, if an obstacle is interposed, a shadow will be cast at P only when the obstacle enters the very narrow cone which has P as apex and the area round O as base. But this area is very small, for taking $\lambda=6\times10^{-5}$ cm. for yellow light, and $r=1$ metre, it has a radius of about ·055 cm., and so may be considered almost as a point. Thus Fresnel satisfactorily explained the rectilinear propagation of light and the formation of sharp shadows on the basis of the wave theory. The great difference between this case and that of sound waves, which are easily diffracted, lies in the magnitude of the wave-length. Taking a corresponding case for sound, $\lambda=4$ ft., or 120 cm. approximately for middle C, so that if $r=1$ metre, as before, the radius of the area around O would be approximately 78 cm. The

difference between the two cases considered is at once apparent.

We shall return to this theory of zones in the chapter on diffraction of light.

Reflection and Refraction at Plane Surfaces.—Let ACB be a plane wave incident on the reflecting surface XY. When A reaches the surface, B has still to travel a distance BD while C has to move CE. While the secondary waves emitted from B and C travel to the mirror, the wave emitted from A starts out from the surface and has a radius vt after a time t.

This distance will be equal to BD by the time the wave from B reaches the surface, and so the new wave front must pass through D and touch a circle of radius BD

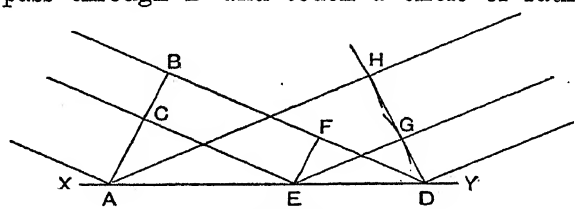


FIG. 150.

drawn about A as centre. In the same time a wave from C has reached E, and a secondary wave will have been set up and will have moved a distance $(BD - CE) = DF$, from E. It is obvious from the diagram that a circle of this radius will just touch the new wave front DH since the ratio of the radii $= \dfrac{DE}{DA} = \dfrac{EG}{AH}$.

Thus for any other point similar to C the secondary wave emitted will just touch this line DH after reflection occurs.

Since $DB = AH$, $B\hat{A}D = A\hat{D}H$, or the angles of incidence and reflection are equal, the angle of incidence being $B\hat{A}D$, the angle between the wave front and the surface.

The treatment is similar in the case of refraction, but here the velocity in the second medium differs from that

in the first. Taking v as the velocity in the first medium and v' as that in the other, then while the wave from B moves to D, that from A moves a distance $\frac{BD \cdot v'}{v}$. The wave front in the second medium will thus be a line DH which passes through D and is a tangent to a circle of radius $\frac{BD \cdot v'}{v}$ described about A. The wave emitted from C will meanwhile have reached the refracting surface at E, and a secondary wave will have been emitted from that point and will have travelled a distance $\frac{FD \cdot v'}{v}$ into the second medium. The new wave front DH will then be a tangent at G to a circle of this radius described about E as centre for $\frac{EG}{AH} = \frac{DE}{AD} = \frac{FD}{BD}$.

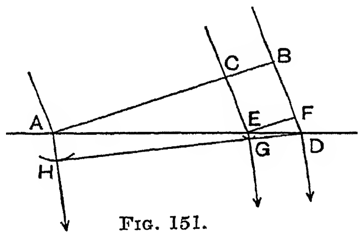


FIG. 151.

So every point on the incident wave front AB gives rise to a point on the refracted wave front DH. Snell's law of refraction may be deduced in this case, where the angle of incidence is $B\hat{A}D$ and the angle of refraction is $A\hat{D}H$.

$$\frac{\sin i}{\sin r} = \frac{BD/AD}{AH/AD}$$
$$= \frac{BD}{AH}$$
$$= \frac{v}{v'} = \text{a constant for the two media.}$$

Now this result is not the same as that obtained on the corpuscular theory, where it was found that $\frac{\sin i}{\sin r} = \frac{v'}{v}$. An experiment by Foucault showed that the velocity in water was less than the velocity in air. This shows that $v > v'$

when $i > r$, thus bearing out the wave theory and overthrowing the corpuscular theory. Since μ for water $= 4/3$ and the value found for $v'/v = \cdot 75$, we are led to the conclusion that

$$\mu = \frac{v}{v'},$$

where v is the velocity in air or vacuo, and v' the velocity in a medium of refractive index μ.

Simultaneous reflection and refraction—so difficult to explain by the corpuscular theory—is easily explained on Huygens' principle that each point on the wave front gives rise to secondary waves. A point P on a wave front which is just in contact with a surface between two media gives rise to waves which are reflected, and others which are transmitted. After a time t the wave in the first medium (air) will have travelled a distance vt while the wave transmitted into the second medium will only have travelled a distance vt/μ.

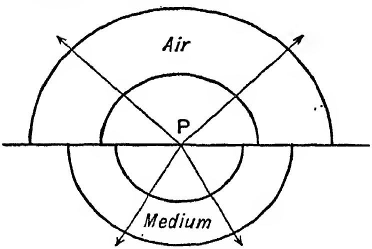


FIG. 152.

Reflection at Spherical Surfaces.—Consider a wave originating from O, a point on the axis of a spherical reflecting surface APB. Let the wave front be ADB when the wave first meets the surface. Now taking the surface to be small compared with the radius of curvature PC—a condition which usually obtains in practice—then while the disturbance at D travels to P, new waves at A and B will be set up and will travel to E and

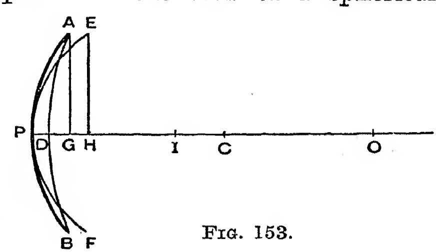


FIG. 153.

F respectively, where AE = BF = PD. The reflected wave front is thus EPF, and converges to a point I on the optic axis.

Draw perpendiculars from A and E to PC. Then AG = EH = x (say). Also if u, v and r equal PO, PI and PC respectively

CONVENTION A	CONVENTION B	CONVENTION C
$DG=\frac{x^2}{2u}$, $PH=\frac{x^2}{2v}$,	$DG=-\frac{x^2}{2u}$, $PH=-\frac{x^2}{2v}$,	$DG=\frac{x^2}{2u}$, $PH=\frac{x^2}{2v}$,
$PG=\frac{x^2}{2r}$	$PG=-\frac{x^2}{2r}$	$PG=\frac{x^2}{2r}$
approximately.	approximately.	approximately.

But
$$\begin{aligned} PH &= PG + GH \\ &= PG + PD \\ &= 2PG - DG \end{aligned}$$

or
$$PH + DG = 2PG.$$

Hence
$$\frac{x^2}{2v}+\frac{x^2}{2u}=2\frac{x^2}{2r}$$

or
$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r},$$

the general formula for spherical mirrors.

Refraction through a Lens.—In the diagram, CDE represents the wave front of a wave emitted from an object O on

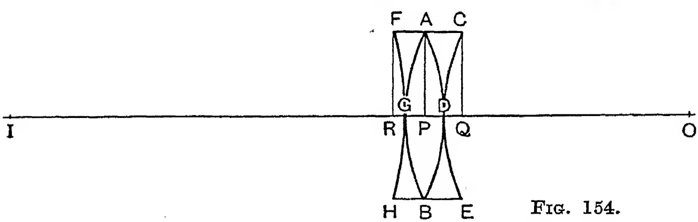


FIG. 154.

the axis of a lens AB. As the wave front at D moves through the centre of the lens other parts of the wave have smaller thicknesses of glass to pass through, and the disturbances at C and E travel to F and H respectively, while that at D

reaches G. So FGH is the transmitted wave front which converges to I. Now the optical paths are equal so that

$$\mathrm{CF} = \mu \,.\, \mathrm{DG}.$$

Draw perpendiculars from A, C and F to the optic axis of the lens. Then taking $\mathrm{AP}(=\mathrm{CQ}=\mathrm{FR})=h$,

CONVENTION **A**

$$\mathrm{QD}=\frac{h^2}{2u},\quad \mathrm{DP}=-\frac{h^2}{2r_1},\quad \mathrm{PG}=\frac{h^2}{2r_2},\quad \mathrm{GR}=-\frac{h^2}{2v},$$

CONVENTION **B**

$$\mathrm{QD}=-\frac{h^2}{2u},\quad \mathrm{DP}=\frac{h^2}{2r_1},\quad \mathrm{PG}=-\frac{h^2}{2r_2},\quad \mathrm{GR}=\frac{h^2}{2v},$$

CONVENTION **C**

$$\mathrm{QD}=\frac{h^2}{2u},\quad \mathrm{DP}=\frac{h^2}{2r_1},\quad \mathrm{PG}=\frac{h^2}{2r_2},\quad \mathrm{GR}=\frac{h^2}{2v},$$

approximately, taking r_1 and r_2 as the radii of curvature of the lens faces ADB and AGB.

Hence $$\mathrm{CF}=\mu \,.\, \mathrm{DG}$$

or $$\mathrm{QD}+\mathrm{DP}+\mathrm{PG}+\mathrm{GR}=\mu(\mathrm{DP}+\mathrm{PG})$$

or $$\mathrm{QD}+\mathrm{GR}=(\mu-1)(\mathrm{DP}+\mathrm{PG}),$$

whence

A:

$$\frac{h^2}{2u}-\frac{h^2}{2v}=(\mu-1)\left(-\frac{h^2}{2r_1}+\frac{h^2}{2r_2}\right)$$

or

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

When $u=\infty,\quad v=f,$

so that

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

B:

$$-\frac{h^2}{2u}+\frac{h^2}{2v}=(\mu-1)\left(\frac{h^2}{2r_1}-\frac{h^2}{2r_2}\right)$$

or

$$\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

When $u=\infty\quad v=f,$

so that

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

C:

$$\frac{h^2}{2u}+\frac{h^2}{2v}=(\mu-1)\left(\frac{h^2}{2r_1}+\frac{h^2}{2r_2}\right)$$

or

$$\frac{1}{v}+\frac{1}{u}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

When $u=\infty,\quad v=f,$

so that

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right),$$

the general equation for lenses.

Dispersion.—Since violet light is more refrangible than red light, it follows that violet light must have a smaller velocity than red light. This is only true when the waves are passing through a medium of refractive index greater than unity, for when light travels through air or a vacuum

($\mu = 1{\cdot}0$) all colours and wave-lengths travel at the same speed.

When a plane wave is refracted through a prism the emergent waves can be obtained as shown in the diagram. ABC is the prism and PQ is the wave front. While the wave from P travels to A the disturbances from Q travel distances $\frac{PA}{\mu_r}$ and $\frac{PA}{\mu_v}$ where μ_r and μ_v are the refractive indices for red and violet light. The wave fronts in the prism are thus AR and AS. These waves travel through the prism, and by the same construction the emergent wave fronts may be found—WX for the red and YZ for the violet light. Thus the violet light is deviated more than the red in the passage through the prism.

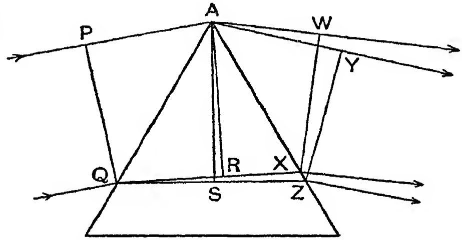


FIG. 155.

Other phenomena which are satisfactorily explained by the wave theory are interference, diffraction and polarisation. These are dealt with in the following chapters. Before these are considered it may be of interest to point out some difficulties which exist at present in the wave theory.

The *quantum theory*, with its conception of light quanta or bundles of energy which travel in straight lines, is not very different from the corpuscular theory with its rapidly moving particles, although the light quanta are not material bodies. Yet it is necessary to apply the quantum theory to explain a number of effects which have so far not been explained satisfactorily by the original wave theory. One instance of this is the photo-electric effect which has already been mentioned in connection with the detection of ultra-violet radiation. The emission of electrons from a metal plate when illuminated depends not only

on the intensity of the light but on its frequency. If the frequency falls below a definite value for that metal no electrons are emitted. The wave theory indicates that energy of light and intensity are proportional, whereas the quantum theory explains the photo-electric effect by postulating that light energy travels in bundles equal to hn, where h is Planck's constant and n is the frequency of the incident light, so that if n is too small there is insufficient energy to liberate electrons.

Another effect which presents difficulties when viewed from the wave theory is the Compton effect. Compton found that when X-rays are scattered by a metal obstacle the wave-length is changed. Scattering of light by tiny particles is a common occurrence, but no wave-length change is expected. On the quantum theory the change is explained by assuming that some of the energy of the light quanta is given to the electrons hit by the quanta. Thus the frequency of the scattered radiation is less than that of the incident rays. Experimental results are in close agreement with those expected by the quantum theory.

EXAMPLES ON CHAPTER XIII

1. What are the reasons for believing that light is propagated in waves and that the wave motion is transverse to the direction of propagation? (Lond. Inter.)

2. Show that the expression

$$y = a \sin \frac{2\pi}{\lambda}(x - vt)$$

represents a train of waves of amplitude a and wave-length λ moving along the x axis with velocity v. Draw curves showing the variation of the displacement y (1) with the time at a point $x = \frac{\lambda}{2}$, and (2) with x at a time $t = \frac{\lambda}{v}$. (Lond. Inter.)

3. Write an essay on "The Development of the Wave Theory."

4. Apply the principles of the wave theory to establish the formula

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$$

for refraction at a spherical surface of radius r separating two media of refractive indices μ_1 and μ_2. (Camb. Schol.)

5. Show that the curvature of a wave emerging from a lens is always equal to the sum of the curvature of the incident wave and the curvature impressed by the lens on a plane wave.

Find the curvature impressed on a plane wave by a double-convex lens, the radii of whose faces are 20 cm. and 25 cm. respectively, the material having refractive index 1·5. (Camb. Schol.)

6. Explain the refraction of light on the undulatory theory, and show that the refractive index is the ratio of the velocities of light in the two media. Indicate (but do not prove) how the formulæ for the refraction of light by spherical surfaces and by lenses can be obtained on this theory. (Camb. Schol.)

7. Apply the principles of the wave theory to find the relation between the focal length of a thin lens, the radii of curvature of the surfaces, and the refractive index of the material.

If a convex lens of focal length 15 cm., made of glass of refractive index 1·52, is totally immersed in a liquid of refractive index 1·35, how will its focal length be affected? (Camb. Schol.)

8. Give reasons for regarding light as a vibration propagated through a medium.

On this theory show that the refractive index of a substance is the ratio of the velocity of light *in vacuo* to its velocity in the substance. (Camb. Schol.)

9. Light from a point source falls on the plane surface of a transparent medium of refractive index μ. Discuss the ensuing phenomena from the point of view of the wave theory of light. (O. & C.)

10. Give an elementary account of the wave theory of light, and prove by its means that the focal length of a concave mirror is equal to half its radius of curvature. (O. & C.)

11. Show how the refraction of light may be explained in terms of the wave theory. What physical meaning has refractive index according to this theory? (O. & C.)

CHAPTER XIV

INTERFERENCE

If two waves of equal amplitude and frequency travel over the same path, the resultant displacement at any point depends on the phase difference between the waves. The two waves superimpose and the resultant displacement at any point along the wave trains is obtained by taking the algebraic sum of the displacements at that point. It is thus possible for the displacement to be zero, if the phase difference is 180°. This occurs for all wave forms—for water waves, for sound waves where compressions due to one wave may exactly balance rarefactions due to another wave, and for transverse waves such as light. Such waves are said to interfere with each other destructively, and the principle of interference has been employed in using Fresnel's half-period zones to show that the rectilinear propagation of light can be deduced approximately on the basis of the wave theory. The effect found there was that the displacements of the waves from one zone balanced the opposite displacements of the waves from another zone.

Although Newton had performed an experiment which is used nowadays to demonstrate interference effects, no thorough explanation was given until the time of Young, who, in the early part of the nineteenth century, established the principle of interference. The chief reason for this long delay was the fact that the corpuscular theory, with Newton's authority behind it, was the accepted theory of light, and it was impossible thus to show how two beams of light could cause darkness.

If we represent two light waves by the expressions

$$y_1 = a \sin 2\pi nt \quad \text{and} \quad y_2 = a \sin (2\pi nt + \epsilon)$$

we have two similar waves differing only in phase. The resultant displacement is found by addition and

$$y = y_1 + y_2 = 2a \sin\left(2\pi nt + \frac{\epsilon}{2}\right) \cos\frac{\epsilon}{2}$$

$$= A \sin\left(2\pi nt + \frac{\epsilon}{2}\right) \quad \text{where} \quad A = 2a \cos\frac{\epsilon}{2}.$$

The new wave is of the same type as the originals, but the amplitude, A, depends on the phase angle ϵ.

$A=0$ when $\epsilon=(2k-1)\pi$, where k is an integer, and $A=\pm 2a$ when $\epsilon=2k\pi$.

So the amplitude varies with the phase difference as shown in the accompanying graph.

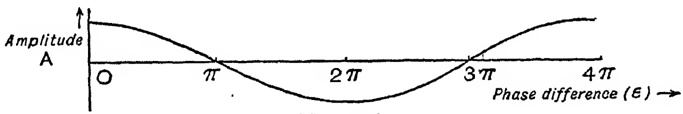


FIG. 156.

It is often more convenient to represent the difference in path which light rays may have, rather than the difference in phase. The expression for the wave is then generally written as

$$y = a \sin\frac{2\pi}{\lambda}(vt + x),$$

where λ is the wave-length, v the velocity of light and x/λ is the fraction of a wave-length travelled before the timing commenced. Substituting $2\pi x/\lambda$ for ϵ in the above equations we find that

$$A = 0 \quad \text{when} \quad x = (2k-1)\frac{\lambda}{2},$$

i.e. when x is an odd number of half wave-lengths.

Similarly the amplitude, and hence the intensity, is a maximum when x equals a whole number of wave-lengths.

Conditions for Interference.—In order to produce visible interference effects it is necessary to have two beams of light which:

(1) originate from a single source of light,
(2) have the same amplitude.

Experiment shows that if two sources of light are used, interference does not result. This is probably due to rapid changes of phase that occur in any source of light owing to collisions between vibrating particles. The amplitudes must be the same, otherwise the one beam has larger displacements than the other, so that although variations in the resultant intensity occur, true interference does not result. If the frequencies are slightly different then the resulting amplitude varies according to the difference in frequency. The corresponding effect is more noticeable in sound when two such waves superimpose, and the phenomenon of "beats" results.

The experiments performed in order to demonstrate interference will now be considered.

Young's Experiment.—As previously mentioned, Young was the pioneer of interference, and in his demonstration of interference effects he illuminated two pinholes, S and S′, with the light from the same source—an illuminated aperture, X—and produced bright and dark bands or

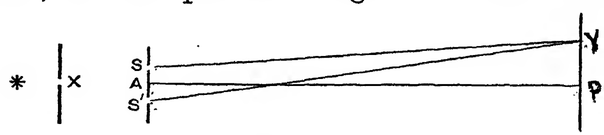


FIG. 157.

fringes on a screen PY. Instead of forming the bands on a screen, a travelling microscope or Ramsden's eyepiece may be used, and the bands observed directly. If P is the central point on the screen, and PA bisects SS′ at right angles, then if PY is parallel to SS′ there will be darkness at Y, provided $S'Y > SY$ by an odd number of half wave-lengths.

Let

$$SS' = 2d, \quad PA = D, \quad \text{and} \quad PY = y.$$

Then

$$S'Y^2 = D^2 + (y + d)^2,$$

and

$$SY^2 = D^2 + (y - d)^2.$$

Hence

$$S'Y^2 - SY^2 = 4yd,$$

or

$$(S'Y - SY)(S'Y + SY) = 4yd.$$

But the interference bands observed are all close to P, and SS′ is small, so that both y and d are very small compared with D.

So we may write

$$S'Y + SY = 2D,$$

and

$$S'Y - SY = \frac{4yd}{2D} = \frac{2yd}{D}.$$

Thus the path difference

$$= \frac{2yd}{D},$$

so that for darkness

$$\frac{2yd}{D} = (2n - 1)\frac{\lambda}{2},$$

while for reinforcement

$$\frac{2yd}{D} = n\lambda,$$

where n is a whole number.

The distance between consecutive positions for reinforcement is evidently $\frac{\lambda D}{2d}$, since for the nth bright point $y = \frac{n\lambda D}{2d}$, and for the $(n+1)$th bright point $y = \frac{(n+1)\lambda D}{2d}$, the difference between the two values of y being $\lambda D/2d$.

If we consider the effect at a point Z on the screen,

where YZ is at right angles to PY, the path difference is the same as before, provided YZ is small. In this case we have

$$(S'Z - SZ)(S'Z + SZ) = 4yd,$$

and again we may take

$$(S'Z + SZ) = 2D$$

giving

$$S'Z - SZ = \frac{2yd}{D}.$$

So if there is darkness at Y there is darkness at points on a line on the screen through Y cutting PY at right angles.

Hence a number of bright and dark bands all perpendicular to PY are produced on the screen, these bands being called interference bands.

The theory of this method is involved in many of the other experiments on interference, and it should be noted that the interference bands produced are equally spaced, whereas diffraction bands, which will be dealt with in the following chapter, are not equally spaced.

A further point to be noticed is that the distance between the bands depends on λ, the distance being greater with red light than with blue light. If white light is used the central band at P is white, the next bright bands, one on each side of P, are almost white but possess slight colouration, while further from P are coloured bands which are rather confused by reason of various wave-lengths overlapping.

Fresnel's Experiments. (a) *Mirrors.*—A single source of light is used as before, but in this case it is an illuminated slit, X, its length being parallel to the line of intersection of two plane mirrors M and N, which are nearly at 180° to each other. An image of X is formed by each mirror, and so we have the equivalent of two slit sources of light at S and S′. The distance between these images can be adjusted by varying the angle between the mirrors. On

the screen are formed interference bands exactly in the same manner as in Young's experiment, and the distance between consecutive bright bands can be measured.

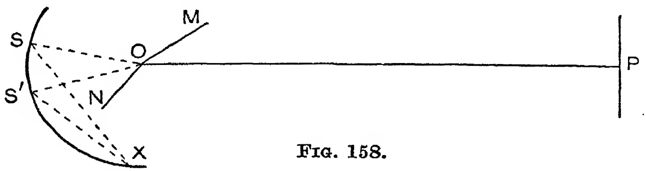


FIG. 158.

Let the obtuse angle between the mirrors be $(180° - \theta)$, and also let $OX = p$ and $(PO + OS)$ or $(PO + OS') = D$. The images S and S′ lie on a circle which has its centre at O and has radius OX. Now the angle subtended by a chord at the centre of a circle is twice that subtended at a point on the circumference.

$$\therefore \ S\hat{O}S' = 2S\hat{X}S'$$
$$= 2\theta,$$

since SX and S′X are perpendicular to M and N respectively.

$$\therefore \ SS' = 2p\theta,$$

and substitution in our former equation for d shows that the distance between successive bands is $\dfrac{D\lambda}{2p\theta}$.

Fresnel's Experiments. (b) *Biprism.*—In this experiment two similar prisms are joined so as to form a single prism, ABCD, in which the angles at A and C are very small and equal, while the angles at B and D are practically

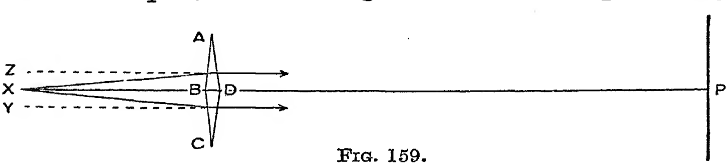


FIG. 159.

180°. Light from a slit X passes through the prism, and owing to deviation caused by refraction, the rays which

pass through the upper half AB of the prism appear to come from an imaginary source of light at Z, while the rays which pass through the lower half of the prism appear to come from Y. Since $A\hat{B}C$ is nearly 180°, Z and Y are close together, and interference effects result, the bands being easily visible through an eyepiece.

To obtain these bands the biprism P is set up with its

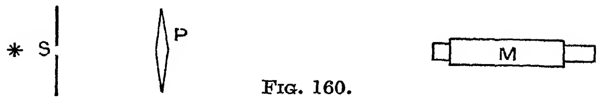


Fig. 160.

edges vertical at the same height as a vertical slit S, which is illuminated by a sodium flame. An eyepiece, with a vernier, or a travelling microscope M may be used to view the bands. (i) This must be arranged in line with the slit and the biprism so that by looking above the eyepiece and along its length two images of the slit are seen, one in each half of the prism. (ii) The slit must then be rotated slowly until the fringes are seen clearly through the eyepiece. In this position the slit is parallel to the ~~axis~~ edge of the biprism. (iii) The eyepiece may now be moved either towards or away from the slit, and the slit width varied until the bands are clearest. The cross-wires are then set on a number of consecutive bands in succession, and the corresponding vernier readings taken. The mean distance x between successive fringes is then obtained from the readings, which should be collected in a table as follows:—

Number of band	Vernier reading	Number of band	Vernier reading	Difference for 5 bands	Distance between consecutive bands
n		$n+5$			
$n+1$		$n+6$			
$n+2$		$n+7$			
$n+3$		$n+8$			
$n+4$		$n+9$			

The distance D from the slit to the cross-wires of the eyepiece is required, and it is also necessary to find the distance between the two images of the slit (formed by the biprism). To obtain the latter a convex lens is placed between the biprism and the eyepiece so that the two images of the slit are in focus when viewed through the eyepiece. If necessary the position of the eyepiece may be changed, but the slit and biprism must be left untouched. The distance between the images seen in the eyepiece is measured by setting the cross-wires on each image in turn and taking the vernier readings. Then, keeping all fixed but the lens, another position is found for this so that the two images are again in focus. The distance between them in the eyepiece is again measured. Suppose the distances are S_1 and S_2 respectively. Then the magnifications are $\frac{S_1}{2d}$ and $\frac{S_2}{2d}$ for the two cases, where $2d$ is the distance required. Now the one magnification must be the reciprocal of the other since only the lens is moved. Thus $2d = \sqrt{S_1 S_2}$.

Hence the wave-length of the light may be found since $\lambda = \frac{2dx}{D}$

Lloyd's Single Mirror Experiment.—By arranging that light from a slit X should fall on a plane mirror MN at practically grazing incidence Lloyd obtained interference bands. The interference occurs between the direct rays from X and the reflected rays which appear to come from S, the image of X formed by the mirror. The slit

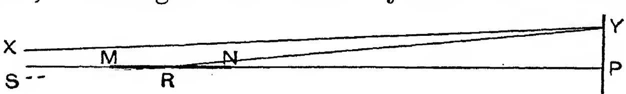


FIG. 161.

is arranged to be parallel to the mirror and perpendicular to the line MP. Since X is slightly above the level of the mirror no reflected rays reach the screen at P, which lies

along MN produced. This means that no central band can be observed. To produce such a band a thin sheet of mica is placed in the path of the direct rays to the screen so that the lengths of the paths XY and XRY may be optically equal. Then a point such as Y is the centre of the system of interference bands. The central band is found by experiment to be dark with bright bands on each side, so that for no path difference the two interfering rays are out of phase. This indicates that the reflected ray must have its phase changed by 180° when reflection occurs.

In the methods described—Young's slits, Fresnel's Mirrors and Biprism—the distance between the fringes depends on λ, so that if white light is used the fringes, apart from the very central ones, are coloured and overlap. By Lloyd's mirror it is possible to produce achromatic fringes, *i.e.* fringes which are successively white and black. This is due to the lateral inversion of the image. If a spectrum is formed at RV by use of a diffraction grating, the reflected image is at R′V′. Then provided we make

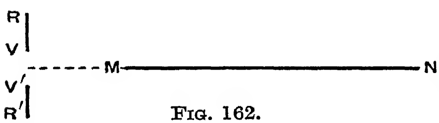


FIG. 162.

$\dfrac{RR'}{VV'} = \dfrac{\lambda \text{ red}}{\lambda \text{ violet}}$, we shall have $\dfrac{\lambda}{d}$ constant for all wave-lengths, since in the spectrum formed by the diffraction grating the dispersion is proportional to the wave-length. Then the distance between the fringes will be the same whatever wave-length we use, for this distance is $\dfrac{\lambda D}{2d}$, and $\dfrac{\lambda}{d}$ is constant. So, if a piece of red glass is placed in the path of the light, red and black fringes result, while, using blue glass, blue and black fringes are caused, the distance between consecutive bands always being the same. Hence, using white light as the source, black and white bands are obtained.

Interference in Thin Films. The Parallel Plate.—Let

AB be a wave front in air incident on one surface of a plane parallel plate of a medium of refractive index μ.

Consider the interference which may result between a ray DC reflected at the front surface and the emergent ray of FD which is reflected internally at F. It should be understood that although the interference of two rays at D only is considered, similar effects occur at all other points on the surface where the incident wave falls. For clearness in the diagram the transmitted rays are omitted. Then the path difference between the rays under consideration is (AF + FD) in the medium, minus BD in air. Now when the wave from B reaches D, that from A will have reached G, where DG is at right angles to AF.

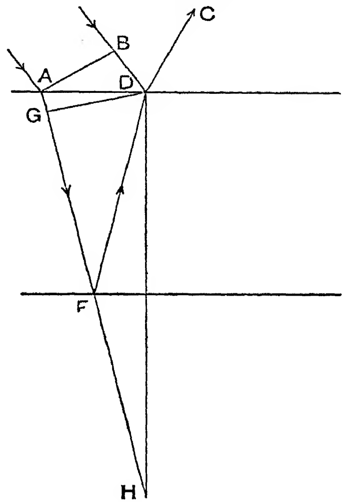


FIG. 163.

Produce AF to H so that FH = AF, and then join HD. Then HD = $2t$ where t is the thickness of the plate, and $G\hat{H}D$ is equal to the angle of refraction θ.

So

$$\begin{aligned}\delta \text{ (path difference)} &= (GF + FD) \text{ in the medium} \\ &= GH = DH \cos G\hat{H}D \\ &= 2t \cos \theta.\end{aligned}$$

This path difference is equivalent to a path difference μ times as great in air. Hence for interference to occur we expect $2\mu t \cos \theta = (2n-1)\frac{\lambda}{2}$, where n is an integer.

Actually a phase change equivalent to $\frac{\lambda}{2}$ occurs at the

reflection at D, and the condition for *darkness* becomes

$$2\mu t \cos\theta = n\lambda,$$

provided that the amplitudes are equal.

Interference between the transmitted rays may also occur. A ray which is reflected twice internally has to travel a further optical path of $2\mu t \cos\theta$ than a ray which is transmitted directly. Here no net phase change occurs, and the condition for *brightness* is

$$2\mu t \cos\theta = n\lambda.$$

Thus the interference fringes seen by reflected light are complementary to those seen by transmitted light.

The Phase Change at Reflection.—Confusion is often caused by the change of phase which occurs on reflection. This phase change occurs only when the ray is reflected at the surface of a medium which is optically denser than the medium through which the ray is travelling. Stokes's analysis of this change is as follows. Consider a ray AB in air incident on a transparent substance of greater refractive index. This ray gives rise to a reflected ray BC, and a refracted ray BD. Let the amplitude of AB be A, of BC be Ax, and of BD be Ay, where $(x+y)=1$. If the reflected and refracted rays are each reversed the resultant should be the initial incident ray. But CB gives rise to a refracted ray BE of amplitude Axy, and DB further increases the amplitude of BE by Ayx'. Since there is no ray BE there can be no intensity of light in the direction BE, and hence no amplitude.

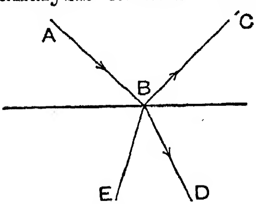


Fig. 164.

Thus

$$Axy + Ayx' = 0$$

or

$$x = -x'.$$

Now x is the fraction of the amplitude reflected at the boundary with a denser medium, while x' is the fraction reflected at the boundary with a rarer medium. These

fractions are numerically equal, but have a difference in sign which denotes that when one ray has a positive displacement the other has a negative one. Hence two rays, one reflected on reaching a denser medium, and the other a rarer medium, are exactly out of phase with each other.

An experiment carried out by Young illustrates this phase change on reflection. If a convex lens is placed on a glass plate and suitably illuminated by monochromatic light a system of concentric rings (Newton's Rings) results when the reflected light is viewed. The central spot is black owing to the phase change at reflection. Young used a lens of crown glass and a plate of flint glass and between them he placed oil of sassafras. This oil has a refractive index less than that for flint glass but greater than that for crown glass. Thus at each reflection—at the lower face of the lens and at the plate—a phase change of π occurs so that there is no net phase change. The experiment showed a white central spot in agreement with the theory.

Colours in Liquid Films.—The effects produced by interference in thin films were first studied by Hooke in 1665. Amongst other phenomena he noticed the rings produced when a convex lens is placed on a plate (Newton's Rings). In practically all cases the reflected light is viewed and the fundamental equation is $n\lambda = 2\mu t \cos\theta$ for interference. If the light reflected from a film of oil on the surface of water or from a soap bubble is viewed it is seen to be coloured. The reason for this is that while t may be approximately constant, θ can have different values since the rays entering the eye are not parallel. Consequently, while there will be reinforcement for certain values of λ, for other values interference will occur and so certain wave-lengths, or colours, will preponderate. For other values of θ other colours will be reinforced so that all the spectral colours will be received by the eye if the incident light is white.

The Wedge.—If two plane pieces of glass are placed in air with their faces at a small angle and are illuminated

by monochromatic light from a slit parallel to the edges in contact, parallel interference bands result, the interference occurring between rays reflected at the surfaces above and below the film respectively. For light incident normally the path difference is $2t$, where t is the thickness of the film at a distance d from the end of the wedge (fig. 165), so that after allowing for the phase change, the condition for darkness is

$$2t = n\lambda.$$

But

$$t = \alpha d$$

and the condition for darkness becomes

$$2\alpha d = n\lambda.$$

The distance between consecutive fringes is evidently $\lambda/2\alpha$, so that if the distance s between the nth dark band and the $(n+k)$th dark band is measured by means of a travelling microscope, α may be found.

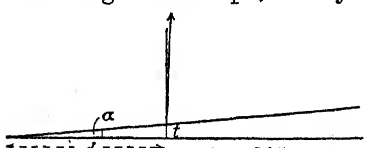


FIG. 165.

For

$$2\alpha d = n\lambda$$

and

$$2\alpha(d+s) = (n+k)\lambda.$$

Hence

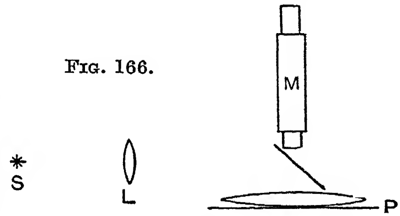
$$2\alpha s = k\lambda.$$

The same equation holds if the transmitted light is viewed.

Newton's Rings.—A case very similar to the wedge is obtained when a convex lens of low power is placed on a glass plate. Using white light as the illuminant, coloured rings, concentric about the point of contact, are formed. They are formed by interference between rays reflected from the plate and rays reflected from the lower face of the lens. These rings are termed Newton's Rings, after Newton, who studied them very carefully without arriving at a definite explanation of their origin. In setting up the apparatus for viewing these rings a cover slip is held at 45° with the horizontal immediately above the glass plate, P. Light from a sodium flame S passes through a convex lens L and is seen reflected from the plate when the eye is held vertically above P. If the low power lens is now

placed on the plate just below the cover slip very small rings will be visible about the point of contact. These may be viewed directly through a travelling microscope M with its axis vertical. Some hundreds of rings will be visible, the distance between consecutive bands getting smaller and smaller the greater the radii of the rings. At the centre of the ring system is a black spot, although if the plate is at all dusty this spot appears rather hazy. At first sight it would appear that this central spot should be bright since there is no path difference between the wave reflected at the lower face of the lens and that reflected by the plate, but it must be remembered that at the former reflection the wave is in the denser medium, while at the latter the wave is in air—the rarer medium. Consequently a phase change equivalent to a half wave-length is introduced.

FIG. 166.



Since we may consider the air film as a parallel plate of gradually increasing thickness the condition for darkness is

$$n\lambda = 2\mu t \cos\theta.$$

But in this case the interference takes place in an air film for which $\mu = 1$, while the light is at normal incidence so that $\cos\theta = 1$. Hence $n\lambda = 2t$, a condition which may be deduced immediately since $2t$ is the path difference between the reflected rays, t being the thickness of the film. If R is the radius of curvature of the lower face of the lens and d is the diameter of the dark ring under consideration,

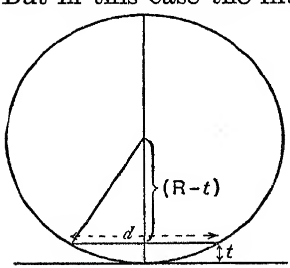


FIG. 167.

$$R^2 = (R - t)^2 + \left(\frac{d}{2}\right)^2$$

or $2Rt=\frac{d^2}{4}$ if we neglect t^2, which is the square of a small quantity. Thus for darkness

$$\frac{d^2}{4R}=n\lambda,$$

showing that $d^2 \propto n$, or the square of the diameter of a ring is proportional to the order of that ring.

In performing the experiment to find λ, a difficulty arises since the exact value of n is not known owing to the haziness of the central spot. In order to overcome this, one of the cross wires of the microscope is set on what appears to be, say, the fifth dark ring, and the vernier reading on the microscope is taken. Readings are taken with the cross-wire set on successive dark rings up to the fourteenth. Further readings are taken for the fifteenth to the twenty-fourth rings, and the experiment is then repeated with the corresponding rings on the other side of the central spot. If the readings are then set out as follows, the mean value for the difference between the squares of the diameters of any ring and the tenth from it may be obtained.

No. of ring (n)	Reading		Diam. d_n	No. of ring ($n+10$)	Reading		Diam. d_{n+10}	$d^2_{n+10}-d^2_n$	$\frac{d^2_{n+10}-d^2_n}{10}=4R\lambda$
	Right	Left			Right	Left			
n				$n+10$					
$n+1$				$n+11$					
$n+2$				$n+12$					
..				..					
..				..					
..				..					
..				..					
..				..					
$n+9$				$n+19$					

If d_n is the diameter of the nth dark ring and d_{n+k} of the $(n+k)$th dark ring,

$$d_n^2=4Rn\lambda$$

and

$$d_{n+k}^2 = 4R(n+k)\lambda$$

whence

$$(d_{n+k}^2 - d_n^2) = 4Rk\lambda.$$

Thus the actual order of the ring is not required. In the experiment described above, $k = 10$, and if R is found ✗ by means of a spherometer, λ may be calculated.

If white light is used fewer rings are seen. The central spot is black and is surrounded by a white ring and then rings of orange, red, violet, blue, green, and yellow in turn.

Refractive Index of a Liquid by Newton's Rings.—If a liquid of refractive index μ is placed between the lens and the plate, the optical path difference becomes $2\mu t$ or $\frac{\mu d^2}{4R}$, using the same notation as before. Then

$$n\lambda = \frac{\mu d^2}{4R} \text{ for the } n\text{th dark ring.}$$

The rings thus obtained are closer together than those formed when air is the medium.

If the diameter of any ring is found with air as the film between the lens and plate, and then the diameter of the same order ring is found with the liquid we have

$$\frac{\text{Diameter using air}}{\text{Diameter using liquid}} = \sqrt{\mu}.$$

Owing to the difficulty of knowing the order of each ring it is best to find the difference between the squares of the diameters of one ring and the tenth ring from that, first using air and second substituting the liquid.

Michelson's Interferometer.—Interference occurs in this instrument between two rays which originate from the same source but pursue different paths before being recombined. Parallel rays of light from a source S are incident on a parallel-sided glass slab A where reflection

and transmission each occur. The rays reflected at the second surface of A fall on a plane mirror M_1 and are reflected back through A in the beam X. The rays which were transmitted through A pass through an exactly

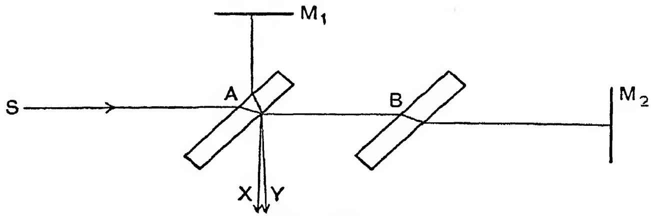


FIG. 168.

similar slab B, are reflected at the plane mirror M_2 and travel back along the same path forming the beam Y after reflection at the second surface of A. X and Y then combine, and interference may result owing to the path difference between these beams. The second glass slab B is included in order to render equal the paths through glass. Each mirror is silvered on its front face and M_1 is mounted so that it may be moved either backwards or forwards. M_2 is fixed in position but may be rotated about either a horizontal or a vertical axis by means of screws. Using monochromatic light, fringes are obtained by adjusting these screws until the mirror M_1 and the image of M_2 in A are parallel. With white light, fringes can only be obtained when each interfering beam pursues the same length of path. The fringes are observed by means of a telescope.

Using this instrument Michelson determined the length of the metre in terms of the wave-lengths of the cadmium red, green and blue spectral lines.

Jamin's Interferometer.—Parallel light is split up into two beams by a glass plate A (fig. 169), some being reflected at the first surface and some at the second. The two beams travel on to a second exactly similar glass plate B where again reflections occur. Taking account only of the reflections shown in the diagram, the resultant beams

X and Y will have travelled equal paths. The other beams which are reflected from B are ignored, since one of them will have travelled only through air while the other will have traversed the plates four times. Both X and Y

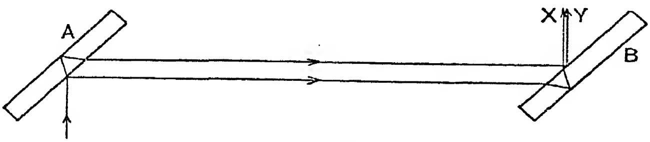


FIG. 169.

have passed through the glass twice. Provided the plates A and B are parallel no interference results, but if B is slightly rotated a very small path difference is introduced and the interference fringes which result may be viewed by means of a telescope.

This interferometer has been used for measuring the refractive indices of gases. Two similar tubes, with glass ends, are evacuated and placed in the paths of the interfering rays between A and B. The gas under examination is then passed slowly into one of the tubes, thus increasing the optical path and causing interference fringes to move across the cross wires of the telescope. If n bands move across, then the optical path is increased by $n\lambda$, where λ is the wave-length. But this increase is due to the increase in refractive index of the gas in the tube.

Hence

$$n\lambda = d(\mu - 1),$$

where d is the length of the tube.

Since the refractive index of a gas depends on the pressure it is necessary to measure the final gas-pressure when the refractive index is being determined.

The Echelon Grating.—This instrument was designed by Michelson and is of importance because it has a very high resolving power (see next chapter). It consists of a number of plates, each of which overlaps the next one by the same distance. The plates all have the same thick-

ness t and are illuminated so that parallel light passes into the largest plate first. Then some of the light passes through air and some through glass, so that a path difference of $(\mu - 1)t$ may be set up at each plate. But this value only holds for light viewed directly through the grating with no deviation, and if the light is viewed to the side then the path difference is greater than $(\mu - 1)t$. Now for reinforcement, $n\lambda = (\mu - 1)t$, so that as the telescope (used for viewing the light) is rotated, positions are obtained for which different values of λ are reinforced. Thus a spectrum is obtained.

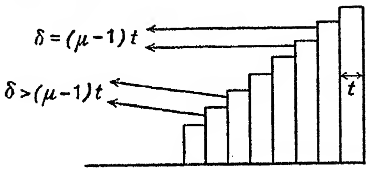


FIG. 170.

Actually a large number of spectra are obtained since a slight change in the position of the telescope gives another position for reinforcement such that $(n+1)\lambda =$ path difference. This is really a disadvantage since the different orders of spectra overlap. In our consideration of the diffraction grating it will be shown that the resolving power equals the product of the order of the spectrum and the total number of lines on the grating. In the echelon grating we consider the number of plates instead of the number of lines. Now the order of the spectrum observed rarely exceeds 2 for the diffraction grating, but for the echelon grating it is in the neighbourhood of 10,000, since t is usually equal to 1 cm. So for an echelon of 50 plates the resolving power is 500,000—a very high value, and it is thus possible to "separate" lines which are extremely close together in the ordinary spectrum. In using the echelon grating, a spectrum is formed in the usual manner and only a small section of this light is allowed to pass through a slit on to the grating. By this means overlapping is eliminated and any part of a spectrum may be examined.

EXAMPLES ON CHAPTER XIV

1. Describe and account for the interference phenomena which can be seen when a convex lens is pressed against a glass plate. Explain, with a diagram, how you would exhibit the phenomena to the best advantage. (Lond. Inter.)

2. Describe the appearance of a wedge-shaped film of air of very small angle confined between glass plates and viewed by parallel reflected light (*a*) when the light is monochromatic, (*b*) when it is white. How may the appearance of the film in the first case be made the basis of a method of determining the angle of the wedge? (Camb. Schol.)

3. Explain the colours seen in a thin soap bubble. (Camb. Schol.)

4. A soap film is formed on a rectangular wire frame held vertically. Account for the coloured fringes which are seen when the film is viewed (*a*) by reflected, (*b*) by transmitted light? (Camb. Schol.)

5. A parallel beam of white light falls normally on a thin air film enclosed between two parallel glass plates, and the transmitted light is brought to a focus on the slit of a spectrometer. A diffraction grating containing 14;468 lines per inch is placed on the spectrometer table and the first-order spectrum is observed. The spectrum is crossed by dark bands, and the deviation of one band from the direct light is $20° \ 35'$, while the deviation for the tenth from this band is $18° \ 50'$. Calculate the thickness of the air film.

6. Explain the interference effects produced when white light falls on a thin film of air enclosed between two parallel glass plates (*a*) if the reflected light is viewed, (*b*) if the transmitted light is viewed.

7. A convex lens is placed on a slab of plane glass and is illuminated by sodium light of wave-length $5{\cdot}890 \times 10^{-5}$ cm. The diameter of the tenth dark ring is measured and is found to be ·406 cm. What is the radius of curvature of the lower face of the lens ?

Using lithium light the diameter of the tenth dark ring is found to be ·433 cm. Find the wave-length of lithium light.

8. Explain the colours of thin films. (O. & C.)

9. Discuss the measurement of wave-length by interference methods, giving examples from various branches of physics. (Camb. Schol.)

10. Under what conditions can interference of light take place? Describe some simple method based on interference for the measurement of the wave-length of light. (O. & C.)

11. Describe some form of interferometer, and explain how it may be used to determine the refractive index of a gas.

12. A vertical slit is illuminated by sodium light, and the light then passes through a biprism, interference bands being observed in an eyepiece. The cross-wires in the eyepiece are set on successive bands which are found to be 0·201 mm. apart. The distance from the slit to the cross-wires is 73·47 cm. When a convex lens is placed between the biprism and the eyepiece, two positions of the lens occur, for each of which two clear images of the slit are seen. The distances between the images are 3·732 mm. and 1·242 mm. respectively. Calculate the wave-length of the sodium light.

13. Explain how achromatic fringes may be obtained by use of Lloyd's mirror.

If a slit, illuminated by sodium light, is placed 2 mm. from the plane of a mirror, what will be the distance between consecutive bands formed on a screen 1 metre from the slit if the wave-length of sodium light is $5{\cdot}890 \times 10^{-5}$ cm.?

14. Show that if two narrow sources are always giving out light in the same phase, and if the light is allowed to fall on a screen placed in a plane parallel to that containing the sources, and if the sources are close together compared with their distance from the screen, then fringes will be obtained on the screen.

Explain how this may be realised in practice. (Camb. Schol.)

15. Under suitable conditions two beams of light may combine to produce darkness over limited regions. What are the necessary conditions which must be satisfied if interference fringes are to be produced? Illustrate your answer by reference to a simple interferometer, such as Fresnel's mirrors. The interfering sources in a simple interferometer are situated d cm. apart, and the fringes produced are observed on a screen D cm. away. Find an expression for the distance between the centres of successive fringes in terms of d and D, and hence show that this distance w is given by

$$w = \lambda/\phi,$$

where ϕ is the angle subtended at the screen by the two sources, and λ is the wave-length of the light used. (Camb. Schol.)

CHAPTER XV

DIFFRACTION

It has already been stated that the acceptance of the wave theory was considerably delayed owing to the difficulty of proving rectilinear propagation by this theory. By the use of zones, Fresnel showed that while light waves travel in approximately straight lines they can bend round corners to a slight extent. Such an effect is termed diffraction. Diffraction phenomena may be conveniently observed by use of a ripple tank in which water waves can be used and the effects noticed when obstacles of varying size are placed in the path of the waves. Also if a plane wave approaches a narrow slit the resultant wave can be seen to be circular, thus bearing out Huygens' theory that each point on a wave front may be considered as a source of secondary waves.

In the case of light waves, diffraction effects may be observed in the following cases:—

(1) A ball-bearing fastened to a piece of plate glass and illuminated by a point source of light casts a circular shadow on a ground-glass screen, the shadow having a small bright spot at the centre.

(2) A wire or needle illuminated by light from a slit (parallel to the wire) gives a number of diffraction bands on the screen.

(3) A straight edge parallel to an illuminated slit causes diffraction bands on to the screen and also shows brightness within the geometrical shadow of the straight edge.

(4) An opaque plate containing a number of circular holes illuminated by light from a circular aperture shows circular diffraction bands on the screen.

In all these cases explanations may be arrived at by

methods similar to that employed in obtaining Fresnel's half-period zones (see page 207).

Circular Aperture.—Let S be the small illuminated aperture and let PQR be the spherical surface due to the waves from S. Let MON represent the screen, and imagine

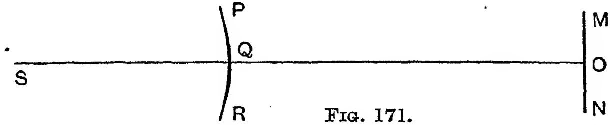


Fig. 171.

spheres of radii $(OQ+\lambda/2)$, $(OQ+\lambda)$, $(OQ+3\lambda/2)$ . . . drawn around O as centre. Now the areas of the rings and the central disc are approximately equal so that the resultant intensity at O is

$$I = s_1 - s_2 + s_3 - s_4 + s_5 - \ . \ . \ .$$

where s_1, s_2 . . . are the intensities due to each zone. As before (page 209), this gives $I = \frac{s_1}{2}$ for a large number of zones. Now supposing that a small obstacle cuts off all the light from the first two zones then the intensity O is $s_3/2$. If only the inner zone is cut off, the intensity at O is $-s_2/2$, so that in both cases there is little loss in intensity. Thus there is a bright spot in the centre of a shadow of a small circular obstacle.

The Zone Plate.—In a manner similar to that just

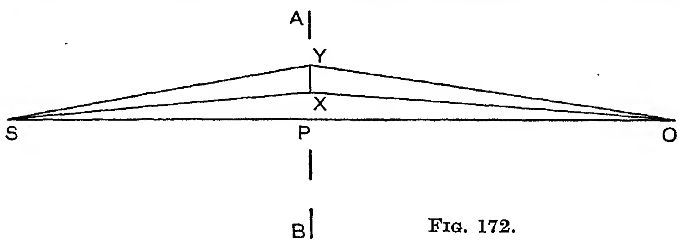


Fig. 172.

employed we can divide a transparent plate AB, perpendicular to the paper, into circular zones of radii r_1,

r_2, r_3 . . . such that wavelets arriving at O from each zone, illuminated by a point source S, will be in phase if they come from the odd zones and exactly out of phase if they originate from the even zones. For this to be so we must have

$$\mathrm{SX}+\mathrm{XO}=\mathrm{SP}+\mathrm{PO}+\frac{\lambda}{2}$$

or

$$\sqrt{a^2+r_1^{\,2}}+\sqrt{b^2+r_1^{\,2}}=a+b+\frac{\lambda}{2}, \quad \text{if SP}=a \text{ and PO}=b,$$

or

$$a\sqrt{1+\frac{r_1^{\,2}}{a^2}}+b\sqrt{1+\frac{r_1^{\,2}}{b^2}}=a+b+\frac{\lambda}{2}$$

or

$$\frac{1}{2}\cdot\frac{r_1^{\,2}}{a}+\frac{1}{2}\cdot\frac{r_1^{\,2}}{b}=\frac{\lambda}{2},$$

so that

$$r_1^{\,2}=\frac{ab\lambda}{a+b}.$$

Also

$$\mathrm{SY}+\mathrm{YO}=\mathrm{SP}+\mathrm{PO}+\lambda$$

giving

$$r_2^{\,2}=2\frac{ab\lambda}{a+b}.$$

Hence

$$r_1^{\,2} : r_2^{\,2} : r_3^{\,2} : : 1 : 2 : 3.$$

Taking s_1, s_2, s_3, . . . as the intensities due to each zone at O, the resultant intensity I is

$$\mathrm{I}=s_1-s_2+s_3-s_4+s_5 \ . \ . \ .$$

where s_1, s_2, s_3 . . . are approximately equal. Consequently if the even zones are all covered the resultant intensity at O is $s_1+s_3+s_5+$ . . ., which is much larger than the intensity obtained when no plate is used. Such a plate with the alternate zones blackened is called a zone plate and has an action precisely similar to that of the convex lens.

The equivalent focal length may be found since the radius of the first zone is given by

$$r_1^2 = \frac{ab\lambda}{a+b},$$

which may be written in the form

$$\frac{1}{a} + \frac{1}{b} = \frac{\lambda}{r_1^2}$$

so that it compares with the lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

For a convex lens forming a real image, v and f are negative so that in this case

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

Hence the equivalent focal length of the zone plate is r_1^2/λ.

If the even zones are coated with a thin film of some transparent substance so that a phase change equivalent to $\lambda/2$ is produced, the intensity at O is doubled and now is $s_1 + s_2 + s_3 + \ldots$

The Slit.—The treatment in the case of a slit is exactly the same as for the circular aperture except that the waves are cylindrical, the axis of the cylinders being the slit itself. The effect of a wire in cutting out the central zones then is to cause an image showing two or more dark lines with a bright line between them. The effect will be fully examined in the case of a straight edge.

Straight Edge Diffraction Bands.—Let S be the illuminated slit and A be the straight edge perpendicular to the paper. Then on the screen the geometrical shadow starts at the point O. Consider the effect on the screen at P where PO $= y$. Join PS, cutting the cylindrical wave surface XAY at B. Then the intensity on the screen at P is due to (1) the half-wave above B; (2) the portion AB which consists of a number of half-period zones.

If this number of half-period zones is even the alternate zones cancel each other out and the effect at P is due only to the half-wave above B—the minimum effect. But if the

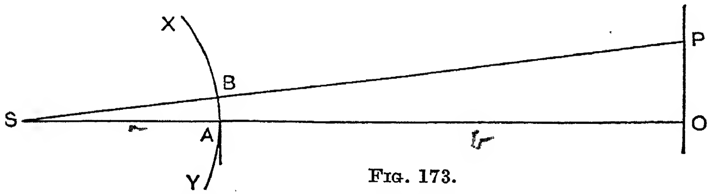


FIG. 173.

number is odd there will be one zone not cancelled and the effect of this at P must be added to that of the half-wave. So as the position of P is changed there will be positions of maximum and minimum brightness, causing diffraction bands. Now the number of half-period zones in AB equals the number of half wave-lengths in the difference of path between AP and BP.

But

$$BP = SP - SB.$$

Hence writing a for SA and b for AO we have

$$BP = \sqrt{(a+b)^2 + y^2} - a$$

$$= (a+b)\left\{1 + \frac{}{(a+b)^2}\right\}^{\frac{1}{2}} - a$$

$$= b + \frac{y^2}{2(a+b)}$$

and

$$AP = \sqrt{b^2 + y^2}$$

$$= b + \frac{y^2}{2b}.$$

Hence

$$AP - BP = \frac{y^2}{2}\left\{\frac{1}{b} - \frac{}{a+b}\right\}$$

$$\frac{y^2 a}{2b(a+b)}.$$

$\therefore$ Number of half-period zones in $AB = \dfrac{y^2a}{b(a+b)\lambda}$

This must be odd in order to give the maximum effect at P.

So for maximum brightness

$$y^2 = \frac{b(a+b)(2n-1)\lambda}{a}$$

and for minimum brightness

$$y^2 = \frac{b(a+b)2n\lambda}{a}$$

where n is a whole number.

The bands thus get closer together as P is moved further from O since $n \propto y^2$.

If P lies within the geometrical shadow, the intensity of illumination gets less and less as P moves down away from O. As the 1st, 2nd, 3rd . . . zones are successively cut off, the intensity at P becomes $-\frac{s_2}{2}, \frac{s_3}{2}, -\frac{s_4}{2} \ldots,$ where $s_1, s_2, s_3 \ldots$ are the intensities due to each zone. Now as P moves downwards the obliquity of the light increases so that $s_1, s_2, s_3, s_4 \ldots$ represent a diminishing series. Hence within the geometrical shadow the intensity of light soon falls away to zero. The complete effect of the straight edge is shown in the graph.

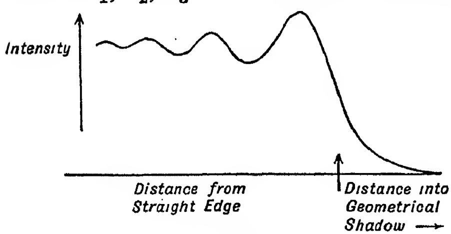


FIG. 174.

If two straight edges are placed close together a slit is formed. Light from one slit falling on another parallel slit is diffracted at the latter, and the intensity on a screen depends on the difference in distance to the point considered from the two edges of the slit. The variation

of intensity with distance from the centre of the slit is shown in fig. 175. Maximum intensity occurs at the centre of the slit, but there are small maxima at the sides.

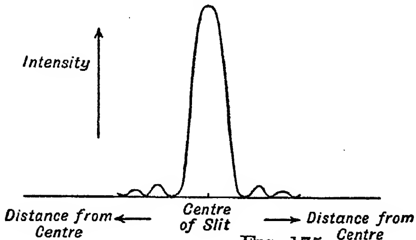


FIG. 175.

The intensities of the second and third maxima are 1/21 and 1/61 respectively of the intensity of the first maximum.

The Diffraction Grating.—If a number of slits, of equal width, are arranged in one plane at equal distances apart, a diffraction grating is obtained. Gratings are made either by ruling a large number of lines at equal distances apart on a piece of glass (the ruling being carried out by means of a fine diamond), or by making a small photographic reproduction of a periodic structure of lines. The number of lines in one inch of the grating is of the order of 20,000. The lines act as obstacles, forming the edges of the spaces between. thus giving us alternate reg

A plane wave front falling on the grating sets up secondary waves at each of the spaces. These secondary waves then either reinforce each other or interfere destructively. Thus in the action of a diffraction grating, interference plays a large part. Let ABCDEF represent a section of a grating, the rulings being perpendicular to the plane of the paper. Imagine a plane wave falling on the grating so that the wave front is parallel to the plane of the grating. Consider a diffracted wave which make an angle θ with the direction of the incident wave. Then BPQR is a

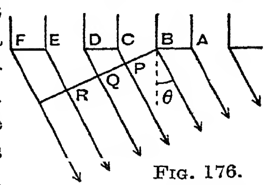


FIG. 176.

part of this diffracted wave front. Now the light from D has to travel a distance DQ further than the light from B, and light from all points in the grating space DE has to travel an equal distance further than the light from corresponding points in the space BC. So when the light is brought to a focus by means of a lens or is received through a telescope, there is brightness or darkness according to this path difference DQ. Let $BC=a$ (the grating space) and $CD=b$. Then $(a+b)$ is called the grating element. From the diagram it is seen that

$$DQ=BD\sin\theta$$

$$=(a+b)\sin\theta.$$

Now for reinforcement $DQ=n\lambda$, where n is a whole number.

Hence

$$n\lambda=(a+b)\sin\theta.$$

Writing d for the grating element we have $n\lambda=d\sin\theta$.

If white light is used a spectrum is formed, since as λ varies so must θ have different values.

When $n=1$ the first-order spectrum results, the second order for $n=2$, and so on.

One of the advantages of the grating is that for the first-order spectrum θ is usually about 20°, so that we can take $\sin\theta$ proportional to θ.

E.g. $\lambda=6\times10^{-5}$ cm. Number of lines per cm. $=6000$.

$$\therefore\ d=\tfrac{1}{6000}\text{ per cm.}$$

For first-order spectrum

$$6\times10^{-5}=\frac{\sin\theta}{6000}$$

or

$$\sin\theta=\cdot36,$$

giving

$$\theta=21^\circ\text{ (approximately).}$$

Hence we have

$$\lambda \propto \theta,$$

or the deviation is proportional to the wave-length.

This fact is made use of when achromatic interference fringes are obtained by means of Lloyd's Mirror.

A serious disadvantage of the diffraction grating is that a large proportion of the incident light passes straight through the grating without any deviation, and this, together with the formation of several spectra, causes a lack of intensity of light in any one spectrum.

Determination of the Wave-length of Light by a Diffraction Grating.—Great accuracy can be obtained in measuring wave-lengths by means of a grating and spectrometer. The telescope and collimator of the spectrometer must be first focussed so that parallel light passes across the spectrometer table. Adjustments now have to be made so that (1) the grating is in a vertical plane, and (2) the ruled lines on the grating are vertical. The slit being illuminated, the reading of the telescope vernier is taken when the collimator and telescope are directly in line so that an image of the slit is formed on the cross-wires. The grating is then placed on the spectrometer table with its plane perpendicular to the line joining two of the levelling screws A and B. The telescope is now turned through 90°, and the table is rotated so that light from the collimator is reflected from the unruled face of the grating and forms an image on the cross-wires. Adjustments of the levelling screws are made until the image is in the centre of the field of view. Then the plane of the grating must be vertical.

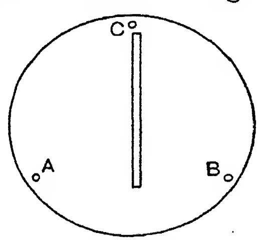


FIG. 177.

The grating is next rotated through 45° so that its plane is perpendicular to the axis of the collimator, the ruled side being towards the telescope. In order to carry out the second

adjustment the diffracted images must be obtained. These are first found by the eye directly. Commencing at the position in which the undeviated light is seen, the eye is moved along in either direction until the first diffracted image is seen. This image is then viewed through the telescope, and the third levelling screw, C, of the table is adjusted in order to bring the image to a central position in the field of view. For this second adjustment the slit should be illuminated by monochromatic light.

The two positions of the telescope for which the first-order images are obtained are recorded, and similar observations made for the second-order spectra. In each case the difference between the readings is halved in order to give the mean angle of diffraction, and the results are recorded as follows:—

	1st order		2nd order	
	Right	Left	Right	Left
Telescope reading .				
Difference in readings .				
θ		$=\theta_1$		$=\theta_2$

The number of lines per centimetre of the grating is also recorded and the wave-length determined from

$$\lambda=d\sin\theta$$

for the 1st order, and

$$2\lambda=d\sin\theta_2/2$$

for the 2nd order.

The third-order images may also be observed, but usually these images are so weak that observations are not trustworthy.

Resolving Power.—When two objects are very close together they may appear as one, and it may be impossible to separate them merely by magnification.

In the case of the eye two objects such as dots appear separate if they subtend at the eye an angle which is greater than about one minute. Optical instruments, especially telescopes, are frequently employed to separate objects which subtend only a very small angle. The separation of such objects is termed resolution. When a spectrum is examined, two lines, close together, are said to be resolved when the maximum of the one is no nearer than the first minimum of the other.

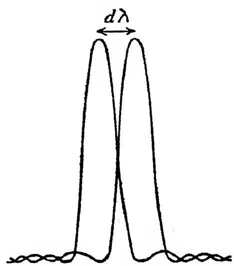


Fig. 178.

The ratio $\lambda/d\lambda$, where λ is the wavelength and $d\lambda$ is the difference in wavelength, is called the resolving power of the instrument used for separating the two lines.

Since resolution is important in certain branches of science, we shall now consider the resolving powers of various optical instruments.

Resolving Power of the Telescope.—Consider parallel rays from two objects falling on the objective of a telescope. Let the angle subtended by the objects be $d\theta$. After

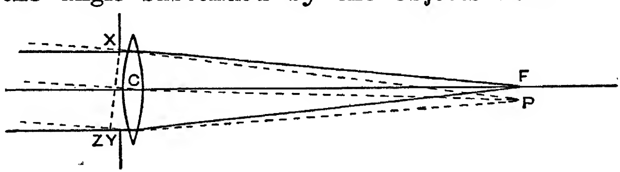


Fig. 179.

passing through the lens the rays from one of the objects converge to a focus at F, while the rays from the other object converge to P. Then $F\hat{C}P = d\theta$, since $d\theta$ is the angle between corresponding rays which arrive at C. Suppose that the objective has a circular stop in front of it, the

diameter of this stop being a. Let the focal length, CF, be f. Then

$$PF = f \,.\, d\theta.$$

Now F is the position for the first maximum of the light from one of the objects, and P the corresponding point for the other object. Also F must be the position of the first mimimum for the latter object when the limit of resolution is reached.

Let XY and XZ be the wave fronts which arrive at the lens. All the wavelets from XY combine to produce maximum brightness at F, while all the wavelets from XZ combine to produce darkness at F. Since rays from Z to F have to travel a distance ZY farther than rays from X to F, then

$$ZY = \lambda.$$

This means that the vibrations at Z and Y are in phase, while for intermediate points on the wave fronts the phases differ by values from 0 to 2π.

But

$$Y\hat{X}Z = d\theta \quad \text{and} \quad XY = a,$$

$$\therefore \lambda = a \,.\, d\theta$$

or

$$\boxed{d\theta = \lambda / a.}$$

This expression gives the limit of resolution of the telescope and shows that for a definite wave-length the resolving power depends on the diameter of the aperture in front of the objective. The greater the value of a, the smaller is the angle which may be subtended for resolution. For this reason the objectives of telescopes have diameters which are as large as possible. It will be remembered that, owing to spherical aberration, only a small part of a lens should be used. In the ordinary camera the lens is stopped down so that only a small central area is employed. Better definition of the image is thus obtained. But in the case of a telescope this would entail a considerable

loss in resolving power, and it is better to use a stop which cuts out the central portion of the objective.

Resolving Power of a Prism.—Let XYZ be the prism. Let AB be a plane wave front of monochromatic light incident on the prism, and let CD be the emergent wave front. Suppose that no stop is used so that light passes through the whole of the prism. Then if μ is the refractive index of the prism for light of this wave-length, λ, we obtain

$$AX + XC = BY + \mu YZ + ZD.$$

If light of a slightly different wave-length, $\lambda + d\lambda$, is incident on the prism at the same angle, the resultant

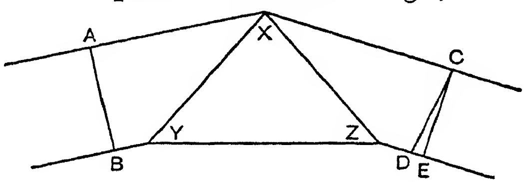


Fig. 180.

wave front is CE, since for a greater wave-length the refractive index is slightly less. Thus, if the refractive index for this wave-length is $(\mu - d\mu)$, we obtain as before

$$AX + XC = BY + (\mu - d\mu)YZ + ZE.$$

A slight approximation is made here by assuming that these rays follow the same paths as the rays of light of wave-length λ.

By subtraction we find $YZ\, d\mu = DE$.

If we write a for the width of the emergent beam, and b for the length of the base of the prism, we have

$$b \,.\, d\mu = a \,.\, d\theta$$

where

$$D\hat{C}E = d\theta.$$

Now the emergent beam is viewed through a telescope for which it has been proved that $a \,.\, d\theta = \lambda$ is the limiting

condition for resolution. Hence for resolution of spectral lines of wave-lengths λ and $(\lambda - d\lambda)$ we must have

$$\lambda = b \,.\, d\mu.$$

Thus the resolving power of a prism

$$= \frac{\lambda}{d\lambda} = b \cdot \frac{d\mu}{d\lambda}.$$

But by Cauchy's formula

$$\mu = A + \frac{B}{\lambda^2},$$

where A and B are constants, so that

$$\frac{d\mu}{d\lambda} \cdot \qquad \frac{2B}{\lambda^3}.$$

$$\therefore \text{ Resolving power of a prism} = -\frac{2Bb}{\lambda^3}.$$

Thus the resolving power of the prism is independent of the angle of the prism and depends only on the length of the base and the material of the prism, and on the wave-length.

Resolving Power of a Grating.—The expression obtained for diffraction at a grating is

$$n\lambda = d \sin\theta.$$

Differentiating this we obtain

$$n \,.\, d\lambda = d \cos\theta \,.\, d\theta.$$

Thus for two waves which differ in length by $d\lambda$, the angle $d\theta$ between the diffracted beams is equal to

$$\frac{n \,.\, d\lambda}{d \,.\, \cos\theta}.$$

The light is received by a telescope so that for resolution $d\theta = \lambda/a$, where a is the width of the beam. Let the

incident light fill the whole of the grating, the width of which is b.

Then

$$a = b \cos \theta.$$

Thus

$$d\theta = \lambda / a = \frac{\lambda}{b \cos \theta}.$$

But

$$d\theta = \frac{n \,.\, d\lambda}{d \,.\, \cos \theta}.$$

$$\therefore \frac{\lambda}{b \cos \theta} = \frac{n \,.\, d\lambda}{d \,.\, \cos \theta}.$$

$$\therefore \text{ Resolving power} = \frac{\lambda}{d\lambda} = \frac{nb}{d}.$$

If N is the number of lines per centimetre of the grating, $d = \frac{1}{N}$, and $\frac{b}{d} = Nb$ = the total number of lines on the grating. Accordingly, the resolving power of a grating is the product of the total number of lines on the grating and the order of the spectrum observed. With the ordinary grating n is only 2 or 3 in general, but with the echelon grating n may have a value of about 10,000. Thus the echelon grating has a very high resolving power.

Simple experiments on resolving power may be made in the case of the diffraction grating. Sodium light is a convenient illuminant since it yields two wave-lengths which differ by about 6 A.U. The grating is fitted up and the first order spectrum is observed. A rectangular aperture is placed on the spectrometer table and the width slowly decreased until it is just impossible to distinguish the two D lines. The second order spectrum is now observed, and it is found that the D lines are resolved for the same width of aperture. The limiting width is again found in this case. It is found to be about half its former

value. The experiment may be repeated for the third order spectrum.

Dispersive Power of the Grating.—It is clearly impossible to use the general formula for finding the dispersive power of a grating, since refractive index is not involved in this case. The greater the dispersion the larger is the angle between the diffracted rays for two given wave-lengths, so that the dispersive power of the diffraction grating is equal to $d\theta/d\lambda$. A value for this may be found directly from the general equation

$$n\lambda = d . \sin\theta \quad \text{. - (1)}$$

From this we obtain

$$n = d . \cos\theta . \frac{d\theta}{d\lambda}$$

or

$$\frac{d\theta}{d\lambda} = \frac{n}{d . \cos\theta}$$

Thus the dispersive power increases with the value of n, *i.e.* with the order of the spectrum. The dispersive power is also larger the smaller the grating element. Finally when θ is large, $\cos\theta$ is small, and a greater value is obtained for the dispersion. This also shows that the red end of the spectrum is drawn out more than the blue, a result which is the opposite of that obtained when the prism spectrum is formed.

Comparison of the Prism and the Diffraction Grating.—
1. The prism concentrates all the light on one spectrum, while the grating produces a number of spectra and also allows much of the light to pass straight through without forming a spectrum.

2. The deviation produced by a prism is not directly proportional to wave-length, and the blue and violet parts of the spectrum produced by a prism are spread out much more than the red part. This is a disadvantage when the energy falling on a given area is required. With the first

order spectrum produced by the grating the deviation is approximately proportional to the wave-length.

3. The resolving power of a diffraction grating is much greater than that of a prism. For a grating of width 5 cm. with 6000 lines per cm. the resolving power is 30,000 for the first order spectrum. For a prism of crown glass, $\mu=1{\cdot}515$ for the hydrogen C line of wave-length 6563 A.U., and for the hydrogen F line, wave-length 4861 A.U., $\mu=1{\cdot}523$. Thus $\dfrac{d\mu}{d\lambda}=\dfrac{\cdot 008}{1702\times 10^{-8}}$ or approximately 500. Taking the base of the prism as 2 cm., we find that the resolving power is 1000—which is just sufficient to resolve the sodium lines. The grating thus has a great advantage over the prism as regards resolving power.

Resolving Power of a Microscope.—Much of the work on the resolving power of microscopes was carried out by Abbe, who showed that the object, illuminated by transmitted light, acted as a diffraction grating. Abbe also showed that the structure of an object could only be observed if the diffracted light was received by the objective of the microscope. In the most favourable case for resolution of two points near together in the object, only the two first diffracted beams (as well as the direct light) must be received by the objective. Then, by the theory of the diffraction grating, we have

$$\lambda=d\sin\theta,$$

where d is the distance between the two points which are just resolved, and θ is half the angle subtended by the

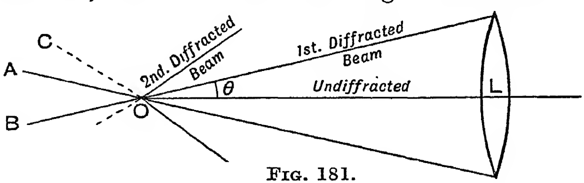


Fig. 181.

diameter of the objective, L, at the object, O. Thus the smallest distance apart for resolution is $\lambda/\sin\theta$. If the

second diffracted orders are visible this value is doubled, since now $2\lambda = d \sin \theta$. Hence the most favourable case occurs when only the two first order spectra are received.

Greater resolving power is obtained if the space between the object and the objective is filled with a transparent liquid of refractive index μ. In this case the effect is as though the wave-length is changed as the light passes from air into the liquid, becoming λ/μ. Hence

$$d = \frac{\lambda}{\mu \sin \theta},$$

showing that a smaller value of d is made possible. Such an arrangement is called an *oil immersion objective*, since oil is most frequently used for filling the space between the object and the lens. Cedar wood oil is generally used, as its refractive index is approximately the same as that of glass. In this case the light received by the lens comes from a wider cone, COD, instead of AOB as before. Then if $\hat{COD} = 2\phi$, $d = \lambda/\sin \phi$. Since $\mu = \frac{\sin \phi}{\sin \theta}$, the expression for d becomes $d = \frac{\lambda}{\mu \sin \theta}$ as before. By using ultra-violet rays instead of visible light the value of λ is smaller, and a smaller value still can be obtained for d. In this case the light is arranged to fall on to a photographic plate after passing through the microscope.

Determination of the Wave-length of Light by Finding the Limit of Resolution of a Telescope.—A piece of fine wire gauze is set up in a vertical plane and is illuminated by a sodium flame. A telescope is placed some distance away, on the opposite side of the gauze to the flame, and is focussed on the gauze. Immediately in front of the telescope is a slit of adjustable width, the edges being parallel to one set of wires in the gauze. Starting with the slit wide open the gauze is seen clearly, but as the slit is narrowed the wires gradually become less distinct, and

ultimately only uniform illumination results (apart from the wires which are at right angles to the edges of the slit). The width of the slit for which the wires parallel to the slit just disappear is measured, a. The mean distance between consecutive wires is also found, d, and the distance of the gauze from the objective of the telescope is measured, D.

Now, for resolution, the smallest angle which may be subtended at the objective $=\lambda/a$, if a is the width of the aperture in front of the objective. But in this experiment the angle subtended by consecutive wires which are just resolved is d/D. Hence

$$\frac{\lambda}{a}=\frac{d}{\mathrm{D}}$$

or

$$\lambda=\frac{ad}{\mathrm{D}}.$$

A number of readings may be taken and a mean value for a/D found.

Instead of a piece of gauze it is better to use a grating on which lines are ruled at approximately 1 mm. intervals.

Coronæ.—When white light passes round small obstacles, systems of coloured fringes are observed. These fringe systems may be seen surrounding the sun and the moon when viewed through a mist or haze. Such rings are termed coronæ, and are caused by diffraction as the light passes through a cloud. They may be imitated by viewing a circular hole, illuminated by white light, through a piece of glass which is lightly dusted with lycopodium powder.

Coronæ only occur in rings of small diameter, and appear to be close to the source of light. The larger rings which are occasionally seen surrounding the sun and the moon are due to refraction as the light passes through crystals of ice in the atmosphere. These larger rings are called *halos*, and appear at some distance from the sun and moon, the angular diameter of the smallest being about 45°.

Scattering of Light.—It has already been mentioned that the extent to which diffraction occurs depends on the wave-length and the size of the obstacle. Blue light is scattered by small objects much more easily than red light, and Lord Rayleigh showed, by the theory of dimensions, that the amplitude of the scattered wave was inversely proportional to the square of the wave-length. Since the intensity (I) depends on the square of the amplitude, then for the scattered wave

$$I \propto 1/\lambda^4.$$

Since the wave-length of red light is almost twice that of blue light, we see that in the scattered wave the intensity of the blue light is 16 times that of the red light.

Thus, as the light from the sun passes through the atmosphere, scattering is caused by the dust particles and, as blue light is scattered most easily, the sky appears blue. When the sun is fairly high the blue of the sky is not intense, and the sun still appears white since only a small distance is traversed by the light in passing through the atmosphere. At sunrise and sunset the light passes a much greater distance through the atmosphere, so that more scattering occurs, the result being that the sky is a deeper blue, while the sun appears red because the blue light is all scattered.

Similar effects are observed when white light is passed through a fog or mist. For this reason orange-coloured lamps are frequently employed for piercing a fog, although these lamps are of little use if the fog particles are large, because then orange light becomes easily scattered. Infra-red rays, with their much longer wave-lengths, will pass easily through smoke or fog.

A simple experiment to illustrate the scattering of light is as follows: White light is passed through a glass tank which contains sodium thiosulphate. A little dilute sulphuric acid is then added. This causes colloidal sulphur to be formed, and the particles gradually increase in size.

As the particles are formed and get larger the tank appears a pale blue when viewed in a direction at right angles to the beam, while the transmitted light becomes yellow and then red before the particles become so large that the light is practically cut off. As this occurs the colour of the scattered light changes from blue to grey.

The same effect may be noticed in tobacco smoke, where the smoke first appears blue when the particles are small, but later becomes grey owing to the increased size of the particles.

EXAMPLES ON CHAPTER XV

1. A parallel beam of white light strikes at right angles a plate in which a very small circular hole is bored, and falls on a screen after passing through the hole. Describe the appearance on the screen, and how it varies as the distance between the plate and the screen increases or decreases.

If the plate with the hole in it were replaced by a small circular disc at right angles to the incident light, what would be the appearance on the screen? What part did the latter experiment play in the controversy which arose as to the validity of the theory of interference? (Camb. Schol.)

2. Describe how you would use the diffraction grating in order to measure the wave-length of the light emitted by a sodium flame.

3. Give a general explanation of the action of a diffraction grating, and explain how, with monochromatic light, a sharp image of a slit is formed. What are the advantages and disadvantages of a grating spectroscope as compared with a prism spectroscope?

4. Parallel sodium light falls normally on a diffraction grating, and the first order spectrum is observed. Two bright lines are observed at deviations $19° \ 39'$ and $19° \ 40' \ 30''$. Calculate the wave-lengths for these lights, the grating having 14,500 lines per inch.

5. When a parallel beam of monochromatic light falls normally on a diffraction grating of m lines per cm. several orders of spectra may be observed, provided the wave-length of the light is not too great. What is the limiting value of the wave-length if the nth order spectrum is just observable (deviation $= 90°$)? Show that by a suitable rotation of the grating it is possible, when the latter

condition has been realised, to reduce the deviation of the nth order spectrum to 60°. How, and in what circumstances, may this change alter the resolving power of the instrument for light of neighbouring wave-lengths? (Camb. Schol.)

6. What measurements must be made in order to determine accurately the wave-length of monochromatic light? (Camb. Schol.)

7. What do you understand by the resolving power of optical instruments? How does it differ from the dispersive power in the case of a prism spectroscope? (Ox. Schol.)

8. Explain what is meant by the limit of resolution of a telescope. A grating which contains lines ruled at 1 mm. intervals is illuminated by sodium light of wave-length $5{\cdot}89 \times 10^{-5}$ cm., and is observed through a telescope in front of which is a slit. The edges of this slit are made parallel to the lines on the grating, and it is found that as the slit is narrowed the lines become indistinct. If the width of the slit is 1·86 mm. when the limit of resolution is reached, calculate the distance of the grating from the telescope.

9. Distinguish between the resolving power and the dispersive power of a grating. On what do these properties depend?

10. Distinguish between the resolving power and the magnifying power of a microscope. What is the use of an oil immersion objective?

11. Find an expression for the resolving power of a prism, and determine the length of base of a prism of angle 60°, which when filled with water will just separate the two sodium lines. The refractive indices of water for light of wave-lengths 6708 and 4340 A.U. are 1·3307 and 1·3404.

12. What are the factors which affect the diminution in intensity which a parallel beam of white light suffers when passing through the air? What advantages are gained by using photographic plates sensitive to light in the infra-red only? What would be the characteristics of photographs taken with plates sensitive only to ultra-violet light or of those taken in a thin fog? (Camb. Schol.)

CHAPTER XVI

POLARISATION AND DOUBLE REFRACTION

IF light is passed through a tourmaline plate, the emergent beam is slightly coloured. When two similar tourmaline plates are placed together in similar positions the emergent light is still only slightly coloured. But if one of the tourmalines is now rotated, the other being kept stationary, the intensity of the beam emerging from the plates decreases until, when the axes of the tourmalines are at right angles, no light is transmitted. As the one plate is further rotated, light reappears and gradually increases in brightness so that after a rotation of 180° the beam appears just the same as at the start.

This experiment shows that the vibrations in a light wave are perpendicular to the direction of motion of the wave, and are not longitudinal vibrations as in sound waves. Longitudinal vibrations take place in the direction in which the wave is travelling, and would not be affected by the rotation of a tourmaline plate about this direction. It is evident that a tourmaline allows only certain vibrations to pass through. By rotating the tourmaline other vibrations pass through, and so we find that in a light wave the vibrations are transverse to the direction of propagation.

The vibrations which are transmitted by a tourmaline plate are all in one direction, and the light is then said to be polarised. To the eye there is no distinction between polarised and unpolarised light. When the tourmalines are parallel this polarised light is transmitted. When they are at right angles the second tourmaline will only transmit vibrations which are at right angles to those transmitted by the first tourmaline. So no light emerges at all. When light is polarised, the plane in which no vibrations occur is called the *Plane of Polarisation*. The

vibrations thus take place in a direction at right angles to the plane of polarisation.]

Polarisation by Reflection.—It was found by Malus in 1808 that the beam of light reflected at the surface of any transparent substance is partially polarised. Further, the degree of polarisation varies with the angle of incidence of the light and, at a certain angle of incidence—depending on the substance—the reflected light is almost completely plane polarised. This angle of incidence is termed the polarising angle. The light is polarised in the plane of incidence, *i.e.* the plane containing the incident and reflected rays.

The vibrations in the incident wave front may be divided into two groups—those in directions in the plane of incidence, and those at right angles to these directions. Now the latter vibrations are parallel to the reflecting surface for all angles of incidence, while the direction of the other vibrations depends on the angle of incidence. When the light is incident at the polarising angle, the vibrations in the plane of incidence are all transmitted so that the reflected light consists only of vibrations at right angles to this plane. The transmitted light is never completely polarised since some of the vibrations at right angles to the plane of incidence are transmitted. The reflected light is almost completely polarised, but the presence of a little dust generally prevents perfect plane polarisation.

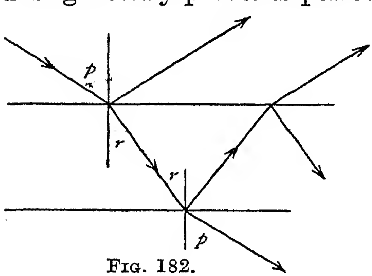


FIG. 182.

Brewster's Law.—Three years after the discovery of polarisation by reflection, Brewster found that a simple relationship existed between the polarising angle p and the refractive index μ of the medium which reflected the light. The relationship is $\mu = \tan p$. This is called Brewster's law. An immediate deduction is that the reflected rays are at right angles to the refracted or transmitted rays.

For

$$\mu = \tan p = \frac{\sin p}{\cos p}$$

$$= \frac{\sin p}{\sin (90^\circ - p)}.$$

But

$$\mu = \frac{\sin p}{\sin r},$$

where r is the angle of refraction.

Hence

$$r = 90^\circ - p$$

or

$$p + r = 90^\circ.$$

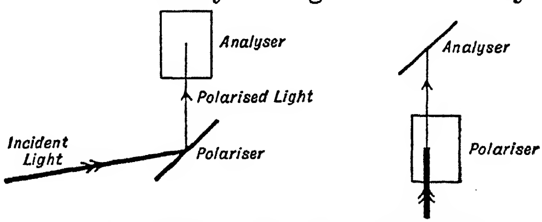
This law is obeyed when light is reflected at the surface of a rarer medium as well as at the surface of a denser medium. Consequently, if light is incident on a glass slab at the polarising angle, the beams reflected from both surfaces will be plane polarised. This affords a very convenient method of obtaining polarised light. But owing to the large percentage of light transmitted by a glass plate the method is not very efficient. This disadvantage may be overcome easily by using a number of glass plates placed one on top of another. Reflection occurs at the upper and lower surfaces of each plate, and since the reflected light is plane polarised—for incidence at the polarising angle—a strong beam of polarised light is obtained. The plates are separated, usually by thin sheets of paper with a central aperture, as otherwise the plates would make good contact with each other and would act practically as a single slab. Such an arrangement is termed a *pile of plates*, and is frequently used for producing plane polarised light. A side arm containing a pile of plates at the correct angle may be fitted to an optical lantern so that a beam of polarised light is produced.

The polarising angle for ordinary glass is about 57·5°.

The value of the angle varies slightly with the wave-length of the incident light since refractive index depends on wave-length.

The fact that the light so reflected is plane polarised may easily be verified. If the light reflected from a pile of plates is viewed through a tourmaline plate, then if the light is incident at the polarising angle, two positions occur for which no light is seen as the tourmaline is rotated through 360°. A single slab of black glass may be used instead of the pile of plates. Black glass is used so that no transmitted light, which is not completely polarised, can be seen.

Instead of examining the reflected light by means of a tourmaline, a second black glass mirror may be used. Both mirrors are usually arranged so that they may be



Front Elevation FIG. 183. Side Elevation

rotated about horizontal axes which are at right angles. Thus the planes of incidence are at right angles. Consequently, if the light incident at $57\frac{1}{2}°$ on the first mirror is reflected on to the second mirror at the same angle of incidence, there will be no resultant beam. This follows since the vibrations in the light falling on the second mirror are in the plane of incidence for this mirror, which only reflects vibrations perpendicular to its plane of incidence. The first mirror is usually called the *polariser*, and the second mirror the *analyser*.

Double Refraction.—A large number of crystals, when illuminated, give rise to two distinct refracted beams. This phenomenon is called double refraction. It was first

noticed by Bartholinus, who in 1669 discovered that when a ray of light is refracted by a crystal of calcite, or Iceland spar, two refracted rays are formed. If a point source of light is observed through a crystal of calcite, two images are seen, and if the crystal is rotated about the line joining the source of light and the eye, one of the images remains stationary while the other moves round with the crystal. The stationary image is called the ordinary image, while the other one is the extraordinary image. The two refracted rays are called the ordinary ray and the extraordinary ray respectively.

Both refracted rays are plane polarised, and their planes of polarisation are at right angles to each other. This may be seen by viewing the images through a tourmaline plate. As the calcite crystal is rotated a position is found for which the ordinary image is at its brightest. For the same position no extraordinary image is visible. When the calcite is further rotated the brightness of the ordinary image decreases, while the extraordinary image reappears. When the angle turned through from the first position is one right angle the ordinary image disappears, while the brightness of the extraordinary image reaches a maximum.]

If a beam of light, plane polarised by reflection at a pile of plates, is passed through the crystal of calcite (without the tourmaline) the changes in brightness of the images may be shown on a screen. The actual planes of polarisation can also be found, since reflected light is always polarised in the plane of incidence. The ordinary ray is polarised in the principal plane of the crystal, the extraordinary ray being polarised in a plane at right angles. The principal plane is the plane which is perpendicular to the refracting surfaces and contains the optic axis of the crystal, this axis being parallel to the line through either of the two blunt corners making equal angles with the three edges there.

The crystal is usually in the form of a rhombohedron,

the six sides of which are similar parallelograms with obtuse angles of 102°.

It was shown by Malus that the intensity of the incident ray equalled the sum of the intensities of the refracted rays. Suppose that the incident light is polarised in a plane which makes an angle θ with the principal plane of the crystal. Then the amplitude of the ordinary ray is $a \cos \theta$, while that of the extraordinary ray is $a \sin \theta$, where a is the amplitude of the incident ray. Since intensity is proportional to the square of the amplitude, we have

$$I_o = a^2 \cos^2 \theta$$

for the ordinary ray, and

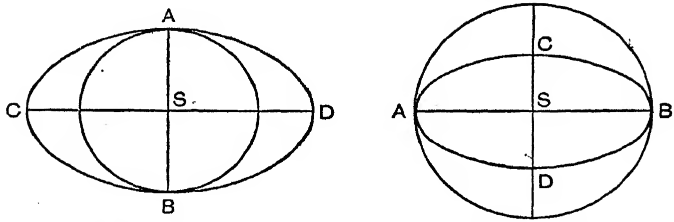
$$I_e = a^2 \sin^2 \theta$$

for the extraordinary ray. Thus

$$\begin{aligned} I_o + I_e &= a^2(\sin^2 \theta + \cos^2 \theta) \\ &= a^2 \\ &= \text{intensity of incident ray.} \end{aligned}$$

Propagation of Light in Crystals.—In all cases of double refraction it is found that the ordinary ray obeys the laws of refraction, while the extraordinary ray disobeys these laws. For the latter ray $\dfrac{\text{sine } i}{\text{sine } r}$ is not a constant. If the incident light is in a direction parallel to the optic axis no double refraction occurs, and the emergent beam is unpolarised. Crystals which have only one optic axis are called uniaxal (*e.g.* calcite). Brewster discovered another set of crystals which have two optic axes and which are called biaxal (*e.g.* selenite). The propagation of light in uniaxal crystals was first examined by Huygens, who found that while the ordinary wave front from a point source S in such a crystal is spherical, the wave front of the extraordinary light is an ellipsoid. Two cases arise—first when the diameter of the sphere equals the minor

axis of the ellipsoid, and second when the diameter of the sphere equals the major axis of the ellipsoid. The former are called negative crystals and the latter positive



Negative Crystal FIG. 184. Positive Crystal

crystals. It is thus seen that for any crystal the velocities of the ordinary and extraordinary rays are different, while the extraordinary rays travel with different velocities in different directions. For the negative crystals, of which calcite is an example, the velocity of the extraordinary ray is greater than that of the ordinary ray, so that $\mu_o > \mu_e$ where μ_o and μ_e are the refractive indices. It is obvious that μ_e is not a constant, but in general the refractive index for the extraordinary ray is measured when that ray is at right angles to the optic axis. In the diagrams AB is the optic axis (the direction along which the rays have equal velocities) and $\frac{\mu_o}{\mu_e} = \frac{CD}{AB}$. Thus in positive crystals, such as quartz, $\mu_e > \mu_o$, and the ordinary ray has a greater velocity than the extraordinary ray.

The Nicol Prism.—One of the most common methods of producing plane polarised light is to pass light through a Nicol prism. This prism is made from a crystal of calcite, the edges of the ends being made equal and about one-third of the length of the crystal. The crystal is cut into two by a plane passing through the blunt corners C and G and parallel to the longer diagonals BD and FH of the ends (fig. 185). The two halves are then joined together again by a thin layer of Canada balsam. The refractive index of

Canada balsam is 1·54, while for the ordinary and extraordinary rays the refractive indices are 1·66 and 1·49. These rays strike the balsam at such an angle that the

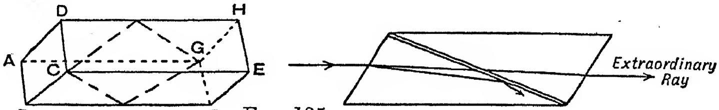


Fig. 185.

ordinary ray is reflected while the extraordinary ray passes through. So the Nicol prism (or the Nicol, as it is often called) produces plane polarised light, the plane of polarisation being at right angles to the principal plane of the crystal.

Nicol prisms are frequently used for producing and for detecting plane polarised light. The prism which produces the polarisation is called the polariser, and the one used for detection is termed the analyser.

A tourmaline crystal is doubly refracting, but the ordinary ray is very weak and is usually absorbed before it passes out of the crystal. The tourmaline thus produces plane polarised light, since only the extraordinary ray is transmitted.

Elliptically and Circularly Polarised Light.—Since, in general, the ordinary and extraordinary rays travel with different velocities, a phase difference is produced between these rays when they pass through a crystal. Consider plane polarised light falling on a plate of a doubly refracting crystal cut with opposite faces parallel. Let the direction of the vibrations in the incident beam be along OP and let the vibrations of the ordinary and extraordinary rays after emergence be OX and OY. Let the amplitude of the incident light be A. Then the amplitudes of the ordinary and extraordinary beams are $A \cos \theta$ and $A \sin \theta$ respectively, where $\theta = P\hat{O}X$.

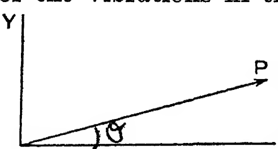


Fig. 186.

In passing through the crystal, suppose that a phase difference ϵ is introduced owing to the difference of the

and dark bands are seen in the field of view. The positions of darkness occur when $\epsilon = 2\pi n$, where n is a whole number, while at the positions of brightness, $\epsilon = (2n-1)\pi$.

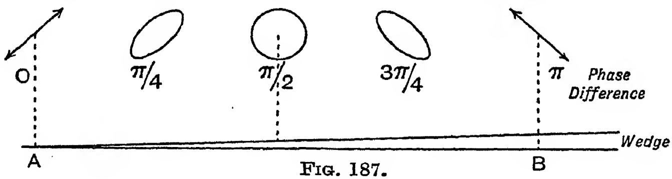


FIG. 187.

The directions of vibration are indicated in the diagram, where it is assumed that the initial vibrations make an angle of 45° with the edge of the wedge. If the Nicols are crossed, the analyser only receives vibrations which are at right angles to those at A, so that A is a position of darkness, while at B there is brightness. Between these points the light is elliptically or circularly polarised as shown. Thus the intensity of light received gradually varies from a maximum at B to zero at A.

When the Nicols are parallel the analyser only receives vibrations parallel to those at A, so that in this case there is brightness at A, and darkness at B.

In the equation $\epsilon = \frac{2\pi d}{\lambda}(\mu_o - \mu_e)$ it will be noticed that ϵ is inversely proportional to λ. This means that the shorter the wave-length of the monochromatic light used the closer together are the bands. Further, ϵ depends on the difference $(\mu_o - \mu_e)$, which also depends on wave-length. In general this variation of $(\mu_o - \mu_e)$ with wave-length is small. If white light is the illuminant the bright and dark bands which would be formed for each separate wave-length overlap, just as they do in experiments on interference with unpolarised light. Consequently, colours are observed through the analysing Nicol, the colour at any point depending on the thickness of the wedge at that point. If the Nicols are initially parallel, and then one is rotated through 90°, the colour at any particular

point changes to its complementary. The reason for this is that in the first case only certain component vibrations were received by the analysing Nicol, while on rotating this Nicol through 90° these vibrations are cut out, and those previously not received are now transmitted.

Instead of using quartz, mica may be employed. Strictly, this is a biaxal crystal, but it closely resembles uniaxal crystals and may be considered as such. If a sheet of mica is placed between Nicols—either parallel or crossed—some colour is observed when the incident light is white. If a second sheet of mica is added to the first the colour is changed since the phase difference between the ordinary and extraordinary rays is altered. Cellophane may also be used for such experiments. For different thicknesses different colours are seen, and in all cases the complementaries are observed when the Nicols are changed from the parallel to the crossed position.

Detection of Plane, Elliptically and Circularly Polarised Light.—If plane polarised light is incident on a black glass mirror arranged so that the angle of incidence is 57·5°, then as the mirror is rotated about an axis parallel to the direction of the incident light, there are two positions in each rotation for which there is no resultant light. If plane polarised light is passed through a Nicol or a plate of tourmaline, then as the Nicol or tourmaline is rotated, extinction of the light occurs twice per revolution. Plane polarised light may also be detected by passing it through a crystal of calcite or other doubly refracting material. As the crystal is rotated, first one image gradually disappears, and for a further rotation this image reappears while the intensity of the other decreases.

When elliptically polarised light is viewed through a Nicol the intensity varies but is never zero. In this, elliptically polarised light resembles partly polarised light. If the light is passed through a quarter-wave plate the axis of which is parallel to either of the component vibrations of the elliptically polarised light, then an

additional phase change of $\pi/2$ is introduced. Thus the total phase difference between the components is now either 0 or π, so that the light is now plane polarised. This may be detected as mentioned in the previous paragraph, and so elliptically polarised light may be distinguished from partially plane polarised light, which does not become plane polarised when passed through a quarter-wave plate.

Circularly polarised light may be detected in a similar manner. When viewed through a Nicol no change in intensity is produced as the Nicol is rotated, so that circularly polarised light gives the same effect as unpolarised light. On passing the light through a quarter-wave plate, the total phase difference between the two vibrations composing the circularly polarised light becomes either 0 or π, showing that plane polarised light is produced. If unpolarised light is passed through a quarter-wave plate it becomes elliptically polarised in general, and so may be distinguished from the circularly polarised light.

Rotation of the Plane of Polarisation.—It is found that certain substances rotate the plane of polarisation of light which passes through them. If two Nicols are crossed no light passes through. But if a solution of sugar is now placed between the Nicols, still crossed, it is found that some light is transmitted. Darkness may be restored by rotating either of the Nicols through some angle. This angle is directly proportional to the concentration of the solution and to the length of path through the solution. It also depends on the wave-length of the light and on the temperature. A substance which rotates the plane of polarisation is termed "optically active." Some substances rotate the plane in a clockwise direction, and others rotate it in the opposite direction to an observer looking towards the source of light. The former are called right-handed and the latter left-handed crystals.

The specific rotation of an optically active substance is defined as the rotation produced when the light passes

through a length of one decimetre of the solution, which contains one grm. of the substance in each cubic centimetre.

Instruments for measuring the optical rotation produced by a substance are called *polarimeters* or saccharimeters. They consist essentially of two Nicols which may be rotated about a common axis. The substance is placed between these Nicols. Since it is difficult to tell accurately the position for perfect darkness, a device is usually included so that the field of view is divided into two halves which can be adjusted to equal brightness. Parallel light is used and the emergent light is viewed through a telescope. One of the devices used for securing two equally

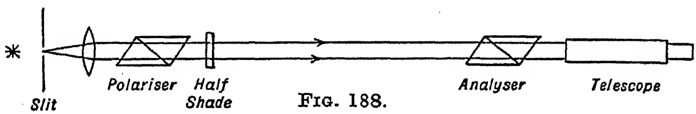


FIG. 188.

illuminated halves of the field of view is the *Laurent half-shade plate*, which is placed immediately behind the polariser. A semicircular plate of quartz ABC is cut so that the optic axis is parallel to the diameter AC. The thickness of this plate is such that a phase difference of π is introduced between the ordinary and extraordinary rays.

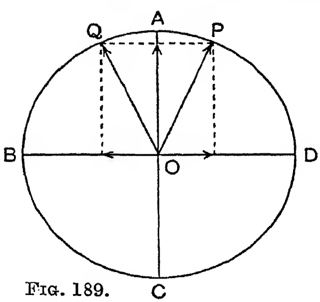


FIG. 189.

A semicircular plate of glass ADC is joined on to this quartz plate, the thicknesses of the two plates being such that equal quantities of light are transmitted. Suppose that the direction of vibration of the light transmitted by the polariser is along OP. This is equivalent to two vibrations, one along OA and the other along OD, at right angles to OA. The phase difference of π introduced by the quartz causes the vibrations to be along OA and OB.

The resultant vibration in the quartz is thus along OQ, where $P\hat{O}A = Q\hat{O}A$, while the vibration in the glass is along OP. Hence in general the two halves of the field of view are unequally illuminated. When the analyser is turned so that the vibrations it transmits are along OA, the two halves appear equally bright. A slight rotation from this position is easily detected. The Laurent plate can only be used for one wave-length, for which the phase difference of π is introduced between the ordinary and extraordinary rays.]

A *biquartz* is sometimes used instead of the Laurent half-plate. Some types of quartz rotate the plane of polarisation in a clockwise direction, while others rotate it to the same extent in the opposite direction—for equal thicknesses of quartz. A biquartz consists of two semi-circular pieces of quartz, one left-handed and the other right-handed, cut so that their faces are at right angles to the axis. These two pieces of quartz are joined together, forming a circular plate, which is placed just behind the polarising Nicol. The two halves of the field of view then appear unequally coloured if white light is used, or unequally illuminated if the light is monochromatic, unless the analysing Nicol is in the correct position. The quartz plates are cut of such a thickness that a slight rotation of the analyser turns one half red and the other blue, if white light is used.

By use of these devices it is possible to adjust the analyser to the correct position with great accuracy. This is of importance, especially with regard to sugar, since it is often necessary in medicine to know accurately the amount of sugar contained in a liquid. The saccharimeter is recognised as one of the best instruments for the estimation of sugars.

In determining the optical rotation of a substance, the position of the analyser for uniform illumination of the field of view is read off when there is only a tube of the pure solvent between the Nicols. A known mass of the

substance is then dissolved in the solvent and the total volume found. When this is placed between the Nicols the analyser is turned until uniform illumination is regained in the field of view. The experiment is carried out several times with different lengths and different concentrations, and a mean value for the specific rotation (s) found from

$$s = \frac{\theta}{l \times c},$$

where θ = angle through which the analyser is turned, l = length of path through the liquid in decimetres and c is the concentration in grams per cubic centimetre of solution.

Sometimes the molecular rotation of a substance is desired. This is found by multiplying the specific rotation by the molecular weight of the substance.

Since the specific rotation of a substance is dependent on the temperature of the substance and on the wavelength of the light, it is necessary to state both these factors in any determination of specific rotation.

Magneto-optical Effects.—In 1845 Faraday, who was searching for relationships between magnetism and light, discovered that when plane polarised light is passed through a piece of "heavy glass" (borosilicate of lead), placed between the poles of a powerful magnet so that the light travels along the magnetic lines of force, the plane of polarisation is rotated. The effect appears to be similar to the natural rotatory effect of a substance such as quartz, but there is a difference between these two as may be seen if the light is reflected back so that it traverses the system again. With quartz the plane of polarisation is rotated back to the original, but in the Faraday effect the reflected light travels along the lines of force in the opposite direction to the previous one, with the result that the rotation of the plane of polarisation is doubled.

Some years later, 1896, Zeeman found an effect for which

Faraday had searched in vain. A sodium flame placed between the poles of an electromagnet was found not to give just the two D lines, but groups of lines. Further, it was found that these components were polarised, and also that the effect was not the same when the direction of the light was along the lines of force as when the light travelled at right angles to these lines.

A further effect was discovered in 1876 by Kerr, who found that plane polarised light when reflected from a pole of an electromagnet became elliptically polarised.

Kerr also found that some transparent substances become doubly refracting when placed in a strong electric field. The difference between the two refractive indices (the ordinary and the extraordinary) is found to depend on the square of the electric intensity.

Use is made of this last effect in television. At the receiving station the varying electric current received is amplified and then arranged to form an electric field in which a Kerr cell, containing nitrobenzene, is placed. On each side of the cell is a Nicol, and a beam of light is passed first through the polariser, then the cell, finally emerging through the analyser. As the field varies so the refractive index of the nitrobenzene changes, and the intensity of the light passing through the analyser is altered just as the light from the different parts of the object alters.

Polarised Light and the Photo-electric Effect. — The emission of electrons from certain metals when illuminated has already been dealt with, and it has been remarked that the electron emission from the metal is chiefly due to the ultra-violet rays. But in a few cases—for the alkali metals —the photo-electric effect results when light from the visible part of the spectrum alone falls on the metal. It has been found that the magnitude of the effect, for the alkali metals only, depends on the condition of the incident light. If this light is plane polarised in the plane of incidence then the emission of electrons is small and increases regularly as the wave-length decreases. When, however, the light is

polarised at right angles to the plane of incidence the photo-electric current increases rapidly as we pass from the red end of the spectrum towards the violet. A maximum is reached for wave-lengths near to the middle of the spectrum, and then for shorter wave-lengths the current decreases quickly. The value of the maximum current is greater the larger the angle of incidence. The wave-length at which the photo-electric current reaches its peak value varies with the metal.

Photo-elasticity.—It is found that ordinary glass becomes doubly refracting when strained. If a slab of glass is placed in a polarimeter and the Nicols are crossed, then the field is dark. As soon as a strain is applied to the glass, by compressing it, the field is no longer dark, thus showing that the glass in its strained condition is doubly refracting. With the removal of the strain the double refraction disappears.

This property of glass is made use of in testing the condition of glass for use in optical instruments. One of the great difficulties in manufacturing lenses is the unequal cooling of different parts of the lenses, thereby causing strains in the glass. Such strains may be immediately detected by placing the lens in a simple polarimeter with crossed Nicols. If the field is not dark then the glass is strained.

A further application is the measurement of strains in buildings. A model of the building is made in xylonite. This substance is a form of celluloid, and has a small elasticity so that it is easily strained. When strained it becomes doubly refracting in the same manner as glass. Instead of celluloid, bakelite and phenolite are often used. These resins have an advantage over celluloid in that the strain vanishes as soon as the stress is removed. With celluloid a permanent strain is frequently incurred. White light is plane polarised by passage through a Nicol, and this polarised light passes through the model to a second Nicol so arranged that the Nicols are crossed.

Absence of strain in any part of the model is indicated by darkness in the field of view. But where the model is strained the field is coloured, and by observation of the colour the degree of strain may be found.

Polaroid.—This polarising material which has recently been introduced has certain advantages over the Nicol for producing and detecting polarised light. It consists of a film of cellulose on which extremely tiny doubly refracting crystals are deposited. These crystals are orientated so that they are all parallel to each other and the film then acts as one large crystal. The crystals are microscopic and are deposited uniformly at about 10^{12} to the square inch. The material then acts in the same manner as a Nicol in that it transmits vibrations in one plane only.

The great advantage of polaroid is that it can be manufactured in unlimited sizes and that it polarises by direct transmission. Thus a disc of polaroid may be placed in front of a projecting lantern and a beam of plane polarised light produced.

There are many possibilities with this new material in commerce. Spectacles made of polaroid suitably orientated can be used in order to cut off or reduce the glare produced by light reflected from polished surfaces. It is only necessary to arrange that the vibrations of the light which is polarised by reflection shall not be transmitted by the polaroid. This elimination of glare is particularly desirable in photography where reflections may obscure details in the photograph.

Another application is the production of stereoscopic effects in pictures and films. Two photographs of the object are taken by means of a stereoscopic camera. These are then projected on to a screen so as to overlap. In front of each projector is a polarising sheet, the polarising axes of the sheets being mutually at right angles. The images are viewed through spectacles containing polaroid glasses suitably set so that one picture only is seen by each eye. A stereoscopic effect is thus obtained.

A further suggested use for polaroid is the elimination of dazzle from motor-car headlamps. It can easily be seen that if each lamp is set to project a beam of plane polarised light, this light is cut off when viewed through a sheet of polaroid, correctly placed, although the car itself remains visible.

EXAMPLES ON CHAPTER XVI

1. Describe how you would produce a beam of plane polarised light, and how you would exhibit its characteristic properties. Give your interpretation of the phenomena you describe.

(Lond. Inter.)

2. Describe the experiments by which you would illustrate what is meant by (1) plane polarised light, (2) circularly polarised light and (3) rotation of the plane of polarisation. (Lond. Inter.)

3. Describe the production and detection of plane polarised and elliptically polarised light.

4. What is meant by polarisation of light? Indicate briefly the principal methods of producing plane polarised light, and describe how you would show experimentally that the light is polarised.

(Lond. H.S.C.)

5. Distinguish between polarised light and natural light. Describe two methods of producing plane polarised light. How would you test whether light is plane polarised or not? (Lond. H.S.C.)

6. Write a short essay on the polarisation of light.

(Camb. Schol.)

7. Describe a method of producing a beam of plane polarised light. How would you distinguish between a beam of plane polarised light and a beam of unpolarised light? Is there any other possibility? (Camb. Schol.)

8. What is the experimental evidence for the theory that light is propagated in transverse waves? (O. & C.)

9. Calculate the thickness of a quarter-wave plate of quartz for light of wave-length 4861 A.U., for which the refractive indices are 1·5590 and 1·5497.

10. Describe the polarimeter.

A sugar solution is placed in a tube 20 cm. long, which is then placed between the crossed nicols of a polarimeter which is illuminated by sodium light. The optical rotation caused by the solution is 10°. If the specific rotation is 67°, find the strength of the sugar solution.

ANSWERS TO EXAMPLES

I (*page* 16)

3. 26·7 cm. 0·56.
4. 81·6 per cent.
5. $5\sqrt{3}$ and 5 foot-candles. 0·35 ft. and 1·46 ft.
6. 16,000 lux.
7. 0·19.
8. 70·4 : 1.
10. 1 : 0·94. 4 : 9.
12. $\frac{16}{25}$.
14. (*a*) 1·25 foot-candles, (*b*) 0·21 foot-candles.

II (*page* 41)

3. 18 cm.
5. 1·414.
6. 1·33.
7. 1·64 cm.
10. 1·6.
12. $\sin^{-1}(\frac{8}{9})$.
13. 1·414.
14. 1·60.

III (*page* 76)

1. 4·17 cm. 25 cm.
4. 52·5 cm.
6. (*a*) 7·5 cm. in front of lens, (*b*) 15 cm. behind lens.
7. (1) $1\frac{1}{11}$ in., (2) $\frac{12}{13}$ in. from lens.
8. 6·8 cm. in front of lens. Real, inverted, diminished.
9. 1·5 cm., 100 cm.
10. 10 cm. from lens.
11. Convex of focal length 9·6 in., 1 ft. from object.
13. (*a*) 10 cm. behind lens; 1 cm. (*b*) 3·33 cm. in front of lens; 0·33 cm.
14. 24·2 cm.
16. 46·9 cm.
17. 6·7 cm. from mirror. 3 : 1.
18. 33·33 cm. from lens. $\frac{4}{9}$.
19. 7·5 cm.
20. (*a*) At infinity, (*b*) 25 cm. from convex lens.
22. 17·5 cm. in front of lens. 0·25.
23. 30 cm.
24. 9·02 from convex lens.
25. 12·9 cm. 12 cm.
26. 5 cm.
27. Between ∞ and $2f$, between $3f/2$ and first lens.
28. 1·33.
29. 15 cm. 1·5.
30. **(A)** $-f(f+a)/(2f+a)$ beyond each lens.
(B) and **(C)** $f(f-a)/(2f-a)$ beyond each lens.
31. 20·625 cm.

IV (*page* 94)

10. 24·0 cm. for flint (concave), 13·34 cm. for crown glass (convex).
11. 37·5 cm. for glass A (convergent), 60 cm. for glass B (divergent).

V (*page* 104)

2. Between lenses and 1 cm. from one lens. 1 cm. beyond other lens.
3. 20 cm. beyond concave, 41·6 cm. beyond convex lens.
4. 5·55 cm.
5. 20 cm., 21·33 cm., 22·86 cm.
7. Principal and nodal points at centre. Focal points 4·5 cm. from centre.
8. 14·91 cm. beyond lens.

VI (*page* 111)

1. 5·28 cm. (convex). 2. Concave lenses of power −1 dioptre, 8·7 cm.
3. 3·75 cm. 4. 4. 2·6 in. from eye.
5. Concave. −10·42 dioptres. 6. 1·5 dioptres. 7. 85·71 cm.

VII (*page* 132)

1. 4 cm. from lens, 5. 2. 9·33.
5. 1·07 cm. from objective, −89. 6. 20.
7. 2·42 cm. 10. 35·33 : 1. 15. 1·28 ft. 2·6.
16. 30′. 18. 2. 19. 4·48. 23. 3 times as large.

VIII (*page* 146)

5. 22° 12′. 6. 50° 31′, 54° 49′. 8. 12 minutes.
9. 52·4°. 11. $\sin^{-1}(\sqrt{\mu^2-1})$. 12. 1·654, 1·646.
13. 25·85°, 5·52°. 16. 22·37°. 17. $2\sin^{-1}\frac{4}{7}$.
18. (*a*) 49° 15′, (*b*) 1° 31′, (*c*) 2·64 cm.
20. 3·98°, 0·035°. 21. 3°, 1·143°.

IX (*page* 164)

8. 34·81 km. per sec. towards earth.

XII (*page* 200)

6. 2500 revs. per min. 8. 3×10^{10} cm. per sec.

XIII (*page* 216)

5. ·045 cm.$^{-1}$. 7. Increased to 61·94 cm.

XIV (*page* 237)

5. $3{\cdot}47 \times 10^{-3}$ cm. 7. 69·95 cm., 6700 A.U.
12. 5890 A.U. 13. 0·0147 cm.

XV (*page* 259)

4. 5889 and 5896 A.U. 5. $\lambda = 1/mn$. Rotation $= 7° 42'$.
8. 3·158 metres. 11. 2·39 cm.

XVI (*page* 279)

9. $1{\cdot}31 \times 10^{-3}$ cm. 10. 7·46 grm. per 100 c.c. of solution.

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